Network Effects

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Why individuals imitate the actions of others?

- Informational effects
  - The behavior of other people conveys information about what they know
  - Thus, observing this behavior and copying it (even against the evidence of one's own private information) can sometimes be a rational decision
- Network effects
  - For some kinds of decisions, you incur an explicit benefit when you align your behavior with the behavior of others.
  - This is what we will consider in this lecture

Network effects

- A natural setting where network effects arise is in the adoption of technologies where interaction or compatibility with others is important
- E.g.: The value of a social-networking or media-sharing site is valuable to the extent that other people are using it as well
- E.g.: A computer operating system can be more useful if many other people are using it
  - Even if the primary purpose of the OS itself is not to interact with others, an OS with more users will tend to have a larger amount of software written for it, and will use file formats (e.g. for documents, images, and movies) that more people can easily read

Positive externalities

- An externality is any situation in which the welfare of an individual is affected by the actions of other individuals, without a mutually agreed-upon compensation
- Example
  - The benefit to you from a social networking site is directly related to the total number of people who use the site. When someone else joins the site, they have increased your welfare even though no explicit compensation accounts for this.
  - This is an externality, and it is positive in the sense that your welfare increases.

Negative externalities

- Traffic congestion is an example in which your use of a (transportation or communication) network decreases the payoff to other users of the network, again despite the lack of compensation among the affected parties.

The market without network effects

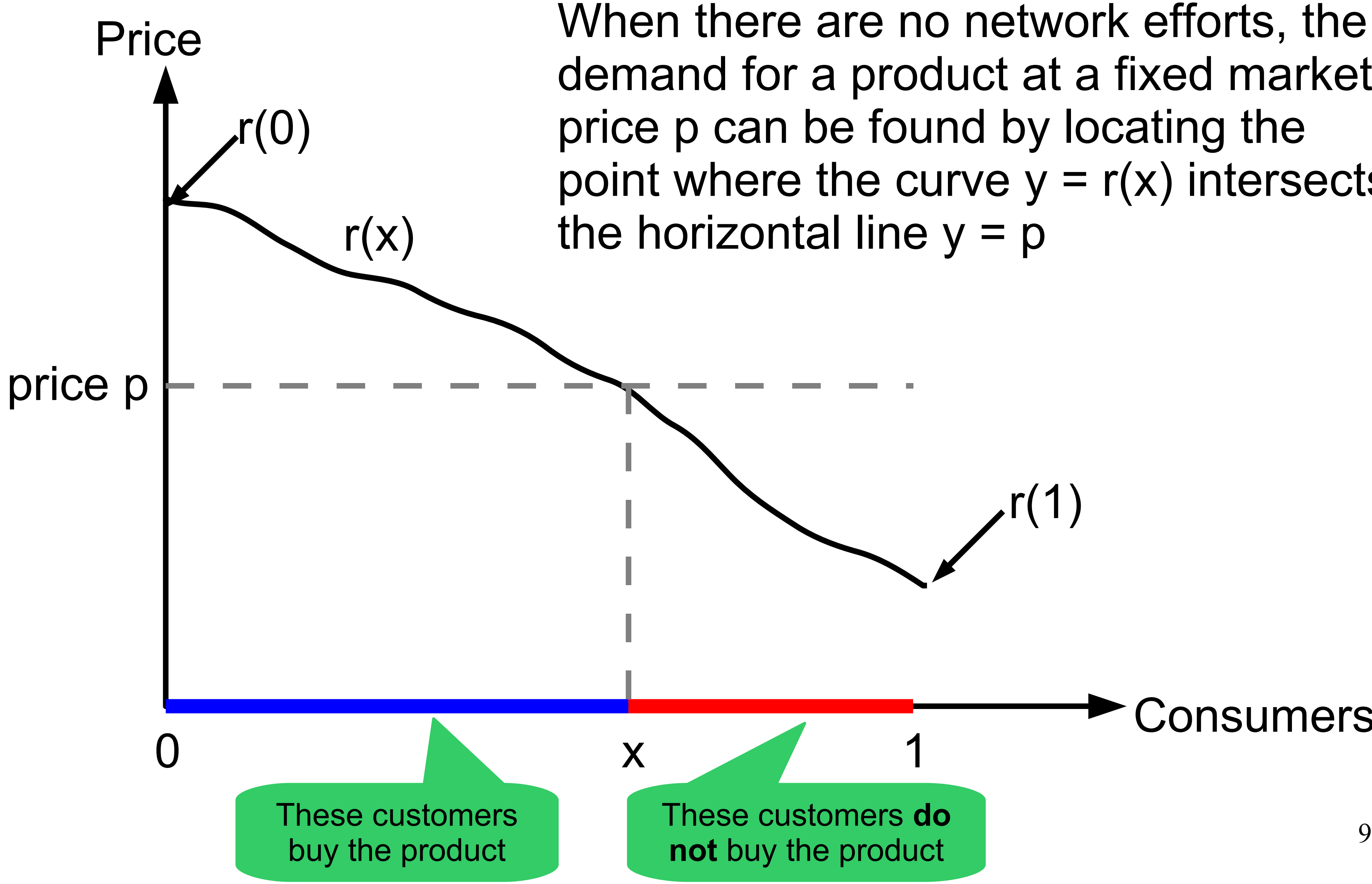
- Let us start to analyze how the market works if there is no network effect
- We assume that each consumer in the market is identified by a real number in the interval [0,1]
  - This approximates the market as a "continuum" of consumers
  - Of course, in real situations there is a large but finite population of consumers
- The interest of consumer *x* in a specific product can be described as the reservation price *r(x)*
  - *r(x)* is the *maximum* price that consumer *x* is willing to pay for buying the product

Reservation price

- Let us assume that r(x) is continuous, and no two consumers have the same reservation price
- Let us further assume that consumers are sorted such that consumer 0 has the highest reservation price, while consumer 1 has the lowest
  - Thus, if $x<y$ then $r(x) > r(y)$
- Assume that the market price for this product is p
  - Each unit is sold at price p

Reservation price

When there are no network efforts, the demand for a product at a fixed market price p can be found by locating the point where the curve y = r(x) intersects the horizontal line y = p



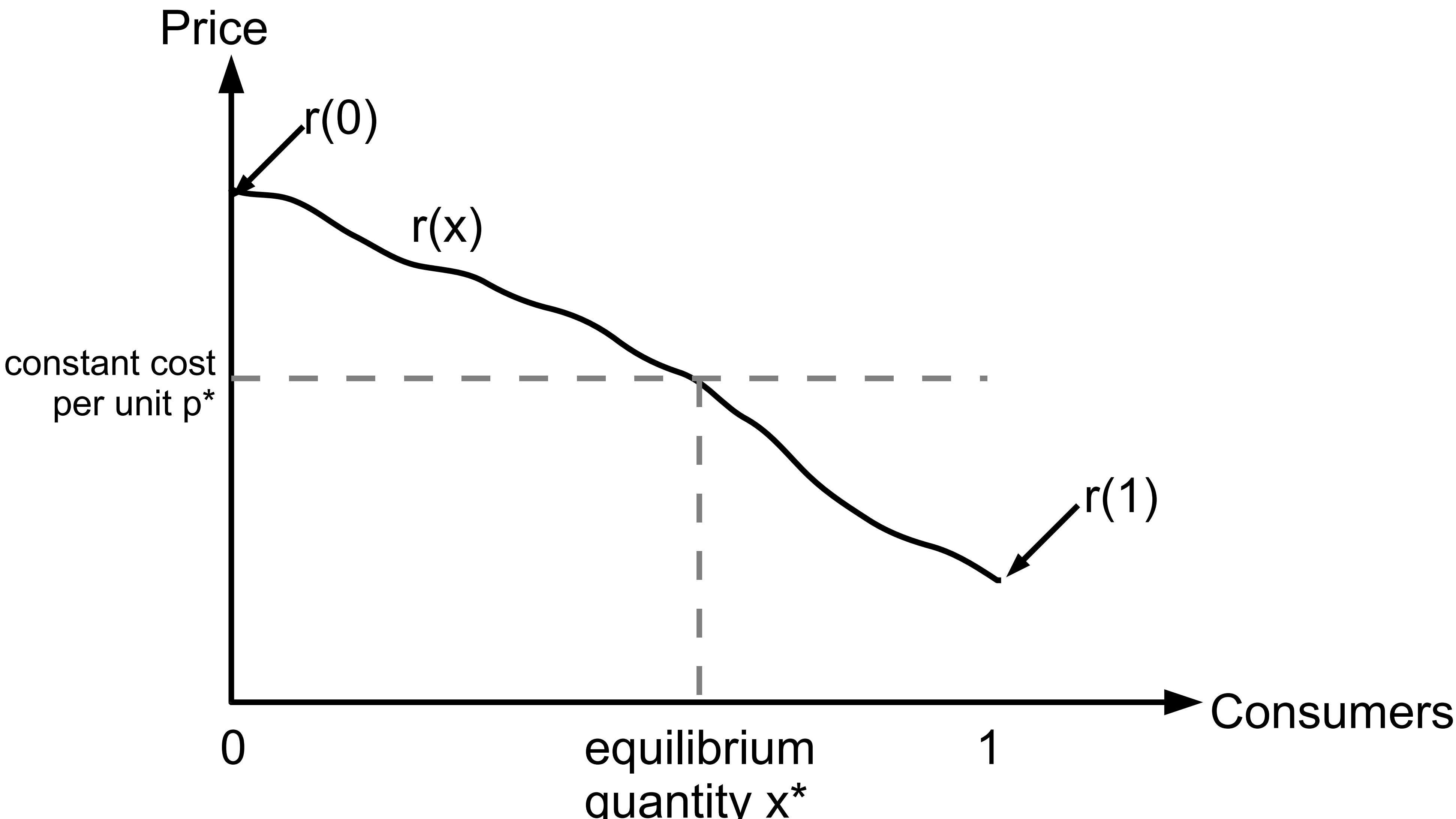
Reservation price

- The reservation price helps us understand how the *demand* of a good works
  - Relation between price and number of units purchased
- Now, let us try to understand how to determine the *supply* of a good

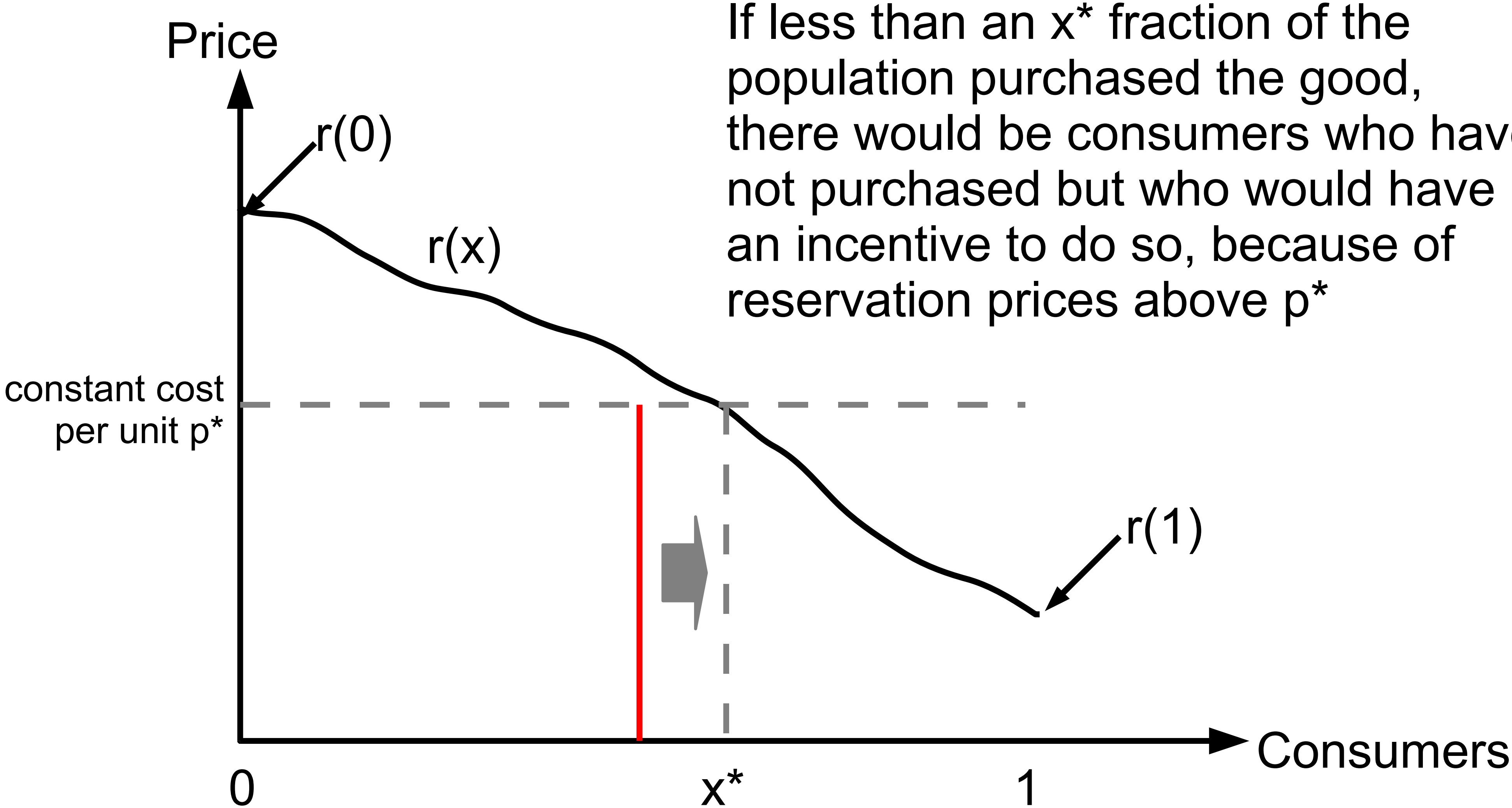
Equilibrium quantity of a good

- Let us suppose that this good can be produced at a constant cost of p^* per unit
- Suppose that there are many potential producers of the good so that none of them is large enough to be able to influence the market price of the good.
  - The price cannot remain above p^* since any profit to a producer would be driven to zero by competition from other producers.
- Then, in aggregate, the producers will be willing to supply any amount of the good at a price of p^* per unit, and none of the good at any price below p^*

Equilibrium quantity



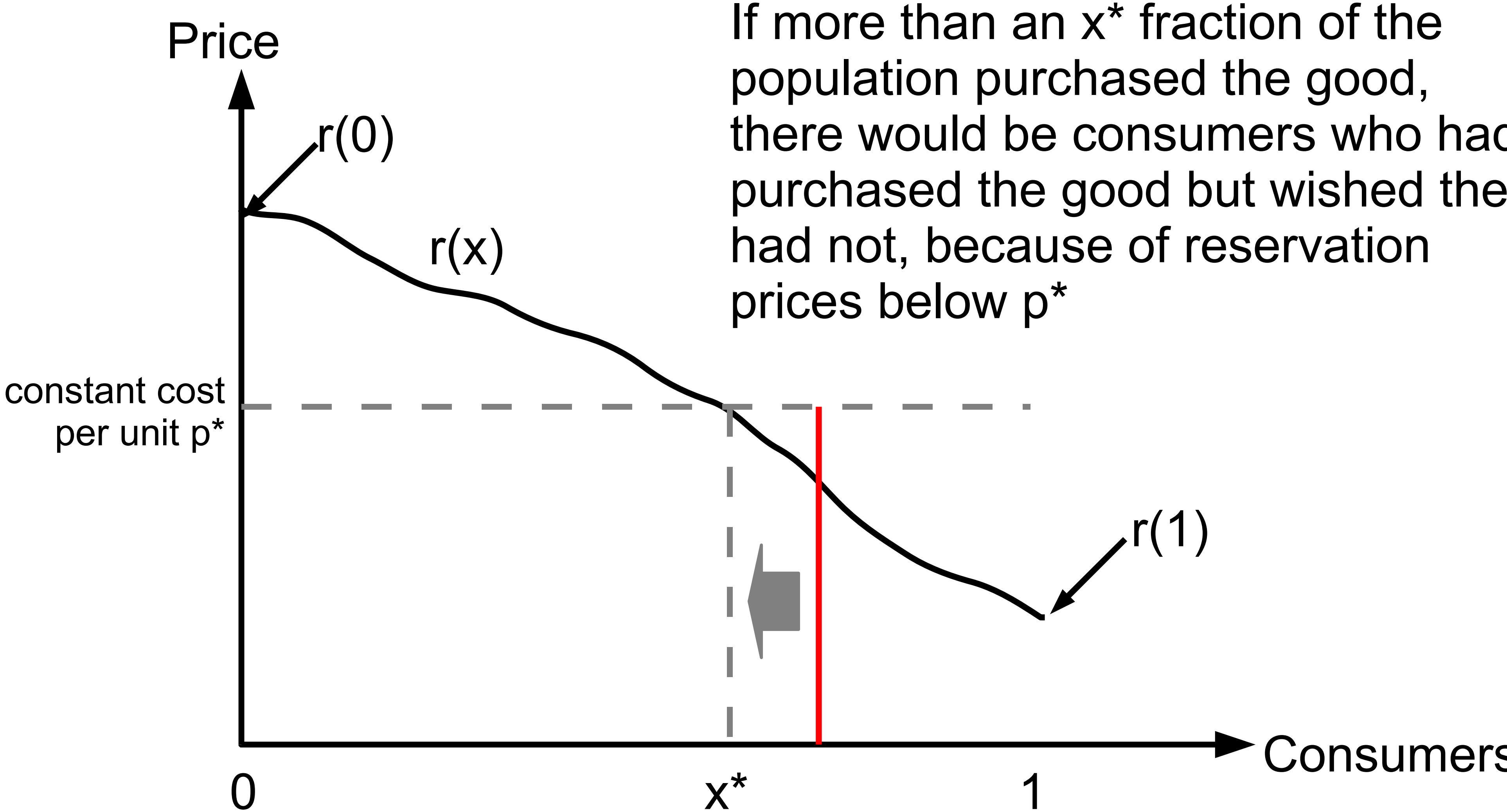
Equilibrium quantity



If less than an x* fraction of the population purchased the good, there would be consumers who have not purchased but who would have an incentive to do so, because of reservation prices above p*

Equilibrium quantity

If more than an x* fraction of the population purchased the good, there would be consumers who had purchased the good but wished they had not, because of reservation prices below p*



The economy with network effect

- If we take the network effect into account, we must consider also the effect induced by the fact that other people are using the produce
- When a z fraction of the population is using the good, the reservation price of consumer x is equal to $r(x)f(z)$
  - $r(x)$ is the intrinsic interest of consumer x in the good
  - $f(z)$ measures the benefit to each consumer from having a z fraction of the population use the good
- The multiplicative form $r(x)f(z)$ means that those who place a greater intrinsic value on the good benefit more from an increase in the fraction of the population using the good

The economy with network effect

- We will assume that $f(0) = 0$
  - if no one has purchased the good no one is willing to pay anything for the good
- We will also assume that f is a continuous function
- We will assume that $r(1) = 0$
  - This means that as we consider consumers x tending to 1 (the part of the population least interested in purchasing), their willingness to pay is converging to 0

The economy with network effect

- Since a consumer's willingness to pay depends on the fraction of the population using the product, each consumer needs to predict what this fraction will be in order to evaluate whether to purchase
- Suppose that
  - the price of the good is p^*
  - consumer x expects a z fraction of the population will use the good
- Then x will want to purchase provided that $r(x)f(z) \geq p^*$

Reservation price of consumer x when a fraction z is using the product

Equilibria with network effects

- What happens when all consumers make *perfect predictions* about the number of users *z* of the good?
- The consumers form a *shared expectation* that the fraction of the population using of the product is *z*
- If each of them then makes a purchasing decision based on this expectation, then the fraction of people who actually purchase *is in fact z*
- We call this a *self-fulfilling expectations equilibrium* for the quantity of purchasers *z*
  - If everyone expects that a *z* fraction of the population will purchase the product, then this expectation is in turn fulfilled by people's behavior

Equilibria with network effects

- Let's consider what such an equilibrium value of z looks like, in terms of the price $p^* > 0$.
  - If everyone expects a $z = 0$ fraction of the population to purchase, then the reservation price of each consumer x is $r(x)f(0) = 0$
- If exactly a fraction z of consumers buys the product, which set of individuals does this correspond to?
  - It will be precisely the set $[0,z]$
- Which is the reservation price?
  - The lowest reservation price p^* is that of consumer z, because for all $x<z$, $r(x)f(z) > r(z)f(z) = p^*$

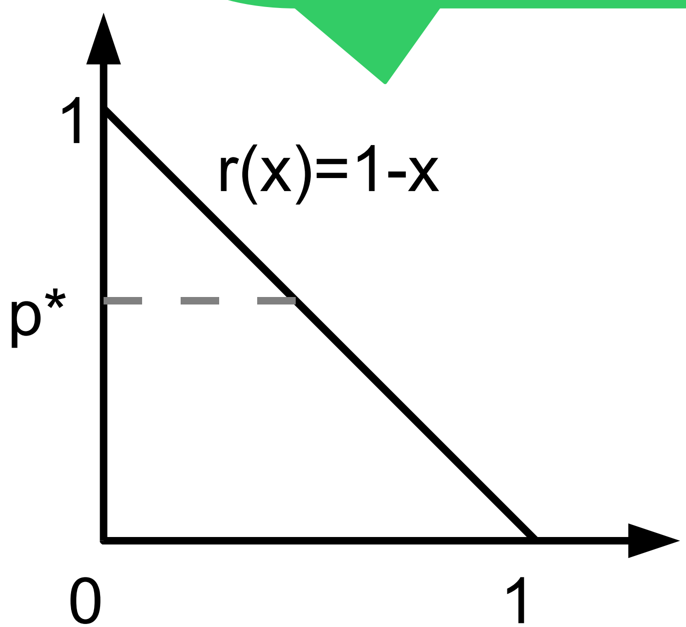
Equilibria with network effects

- If network effects are NOT present
  - In order to sell more products, the price has to be lowered
- If network effects are present
  - If the price $p^* > 0$ together with the quantity z (strictly between 0 and 1) form a self-fulfilling expectations equilibrium, then $p^* = r(z)f(z)$
  - The dependency between price p^* and number of consumers z is more complex

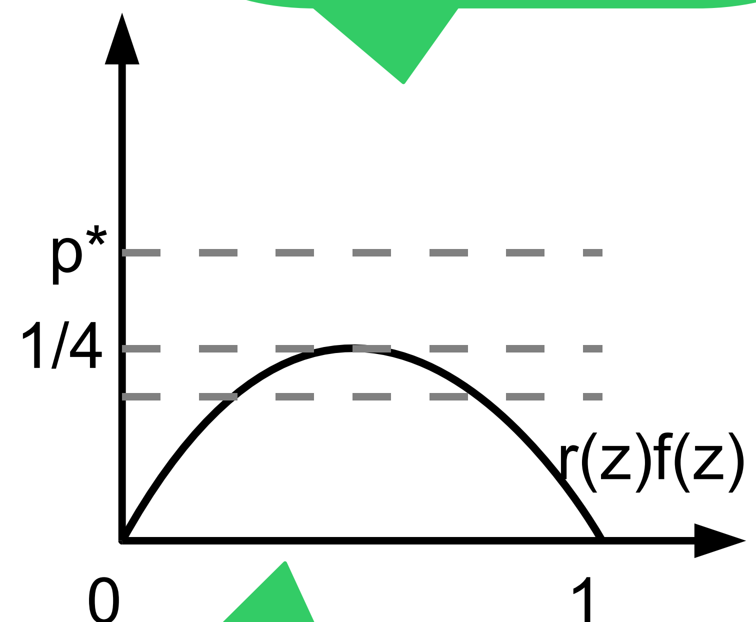
Numerical example

- Let us assume r(x) = 1-x, f(z) = z

The price p* can be anything from 0 to 1



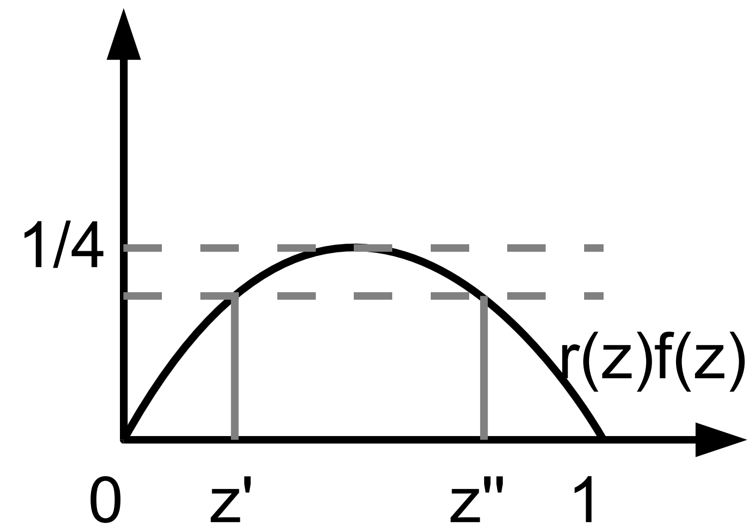
The price p* must be lower than 1/4



Note that if p* < 1/4, there are multiple self-fulfilling expectation equilibria

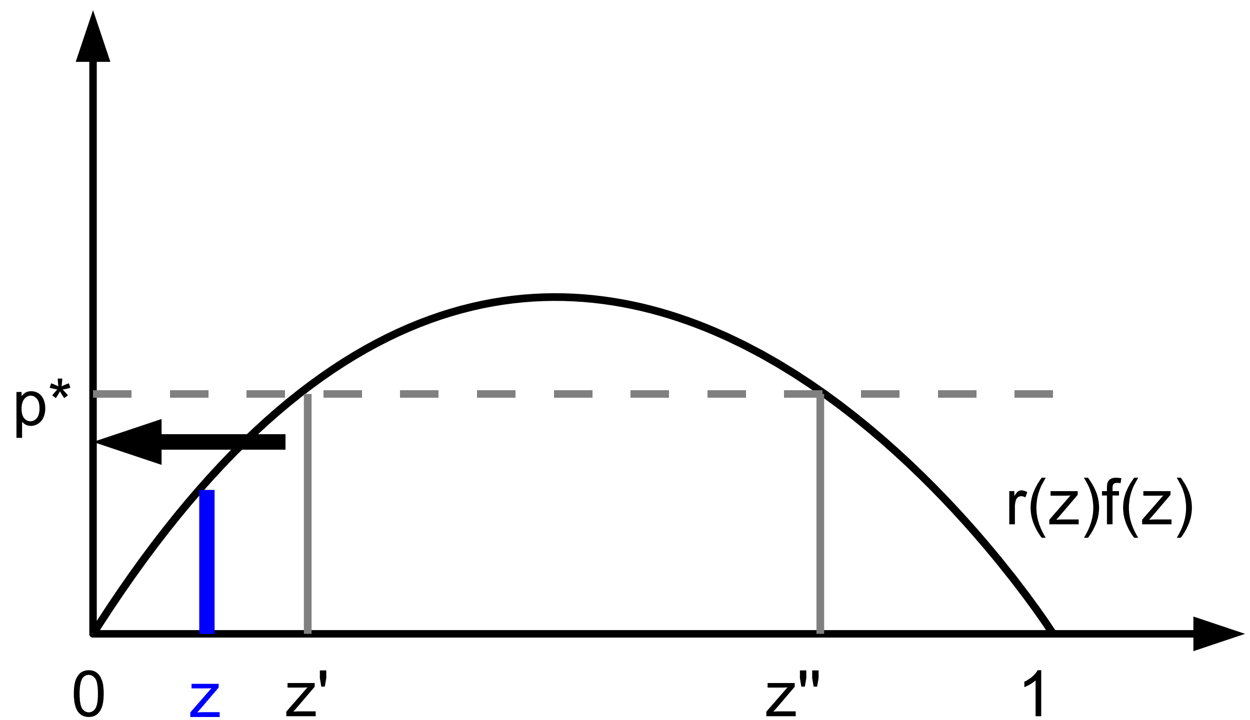
Observation

- As the price p* drops below 1/4, there are two equilibria:
  - z' moves towards 0
  - z'' moves towards 1
- If consumers are confident of the success the the product, they will likely buy it
  - Equilibrium z''
- If they are NOT confident, nobody will buy it
  - Equilibrium z'



Observation: the only equilibria are 0, z' and z''

- Suppose that only a fraction z < z' decides to buy
  - Since r(z)f(z) < p* the consumer named z (and those just below him) will value the good at less than p* and thus they wish they had **not** bought it

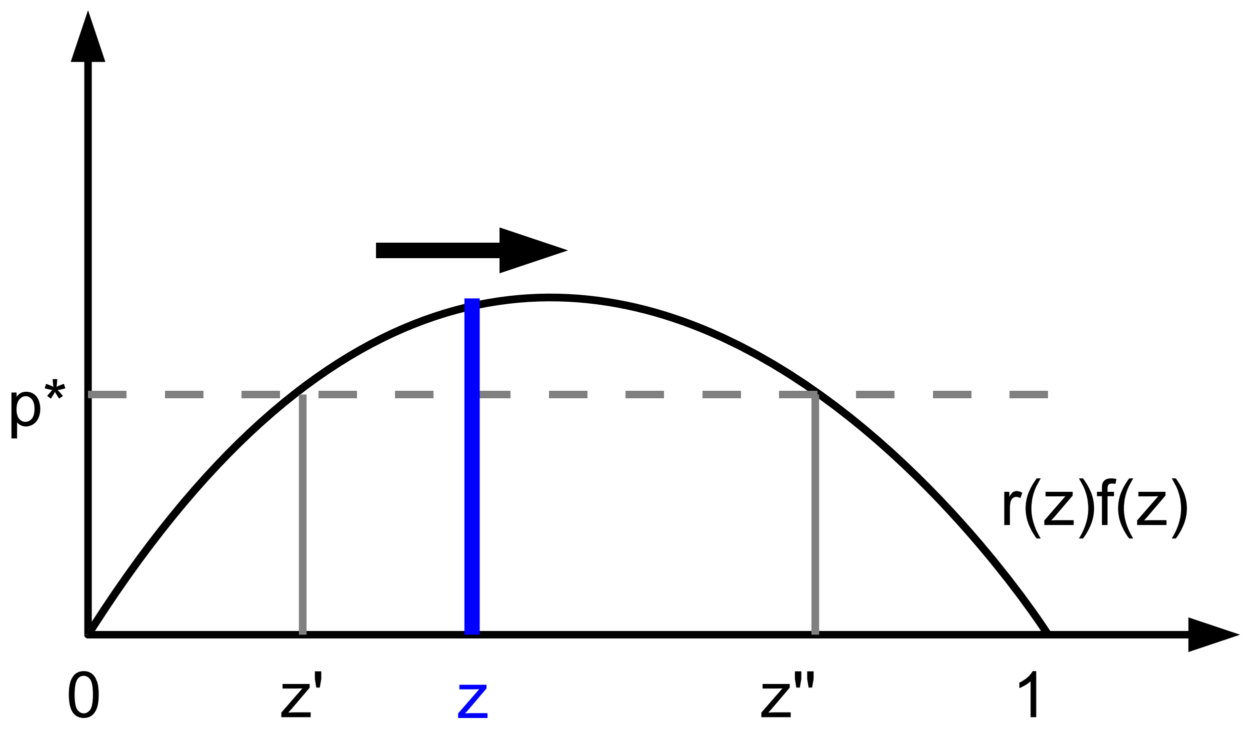


Observation: the only equilibria are 0, z' and z''

- Suppose that only a fraction z < z' decides to buy
  - Since r(z)f(z) < p* the consumer named z (and those just below him) will value the good at less than p* and thus they wish they had **not** bought it
- Why?
  - Consumer x is willing to pay r(x)f(z)
  - We know that r(z)f(z) < p*, because z buys
  - We assume that r() is strictly monotone decreasing
  - So, for customer x = z-ε we have r(x) = r(z-ε) > r(z)
  - We assume that r() is continuous, so for sufficiently small values of ε we have r(z)f(z) < r(x)f(z) < p*

Observation: the only equilibria are 0, z' and z''

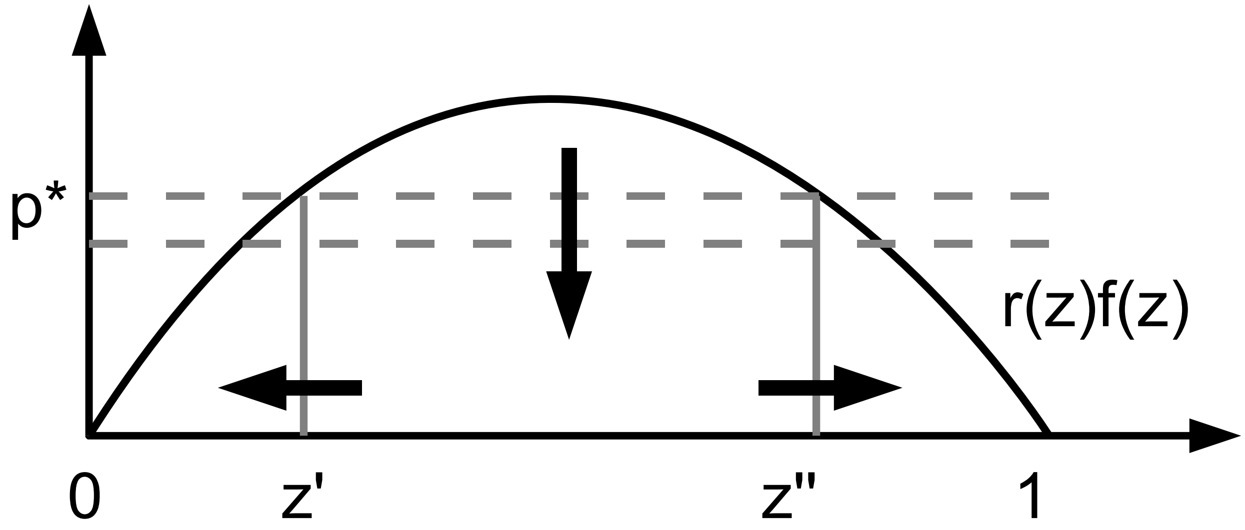
- Suppose that a fraction z' < z < z'' decides to buy
  - Since r(z)f(z) > p* the consumer named slightly more than z will still value the good at more than p* and thus they wish they had bought it



Tipping point

- If slightly more than z' buy the good, then there is an "upward" pressure towards the equilibrium point z''
- If slightly less than z' buy the good, there is a "downward" pressure towards the equilibrium 0
- If the firm producing the good can get the population's expectations for the number of purchasers above z', then they can use the upward pressure of demand to get their market share to the stable equilibrium at z''

Tipping point



- If the firm were to price the good more cheaply (lower p*) then this would have two beneficial effects.
  - the low equilibrium z' would move left; this provides a critical point that is easier to get past
  - the high equilibrium z'' would move right, so if the firm is able to get past the critical point, the eventual size of its user population z'' would be even larger

Tipping point

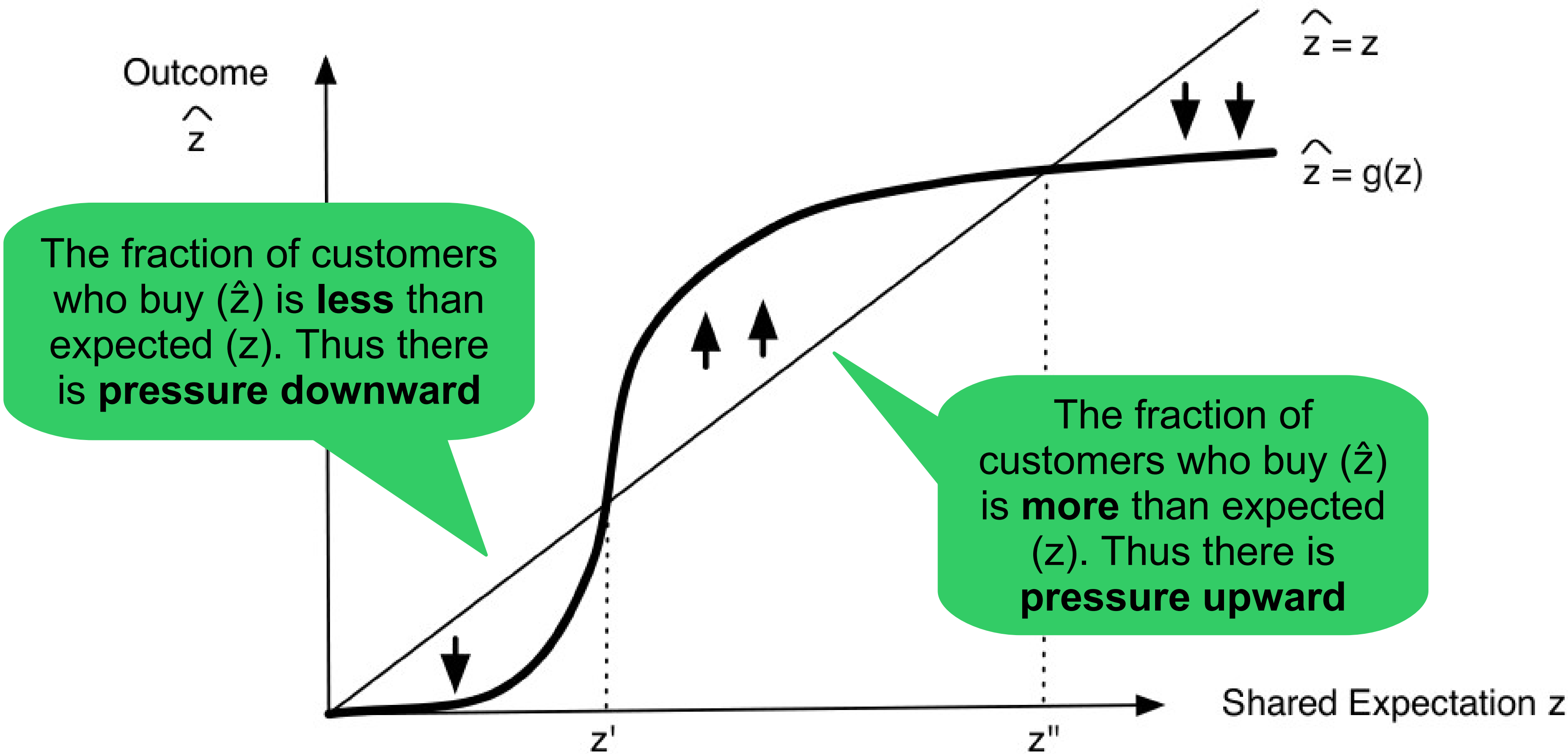
- Now we try to reformulate the problem, assuming that all consumers have a common belief of how many people will buy the product, but this belief is NOT correct
  - If everyone believe a z fraction of the population will use the product, the consumer x will want to purchase if $r(x)f(z) \geq p^*$
  - Hence, the set of people who will purchase will be between 0 and $\hat{z}$, where $\hat{z}$ solves the equation $r(\hat{z})f(z) = p^*$, or

$$r(\hat{z}) = \frac{p^*}{f(z)} \quad \Rightarrow \quad \hat{z} = r^{-1}\left(\frac{p^*}{f(z)}\right)$$

Tipping point

- In general we can define a function $\hat{z} = g(z)$ which gives the outcome $\hat{z}$ (number of people which actually buy the product) in terms of the shared expectation z ("expected" number of people which supposedly buy), given the price p*

Tipping point



From: D. Easley, J. Kleinberg. *Networks, Crowds, and Markets: Reasoning About a Highly Connected World*. To be published by Cambridge University Press, 2010

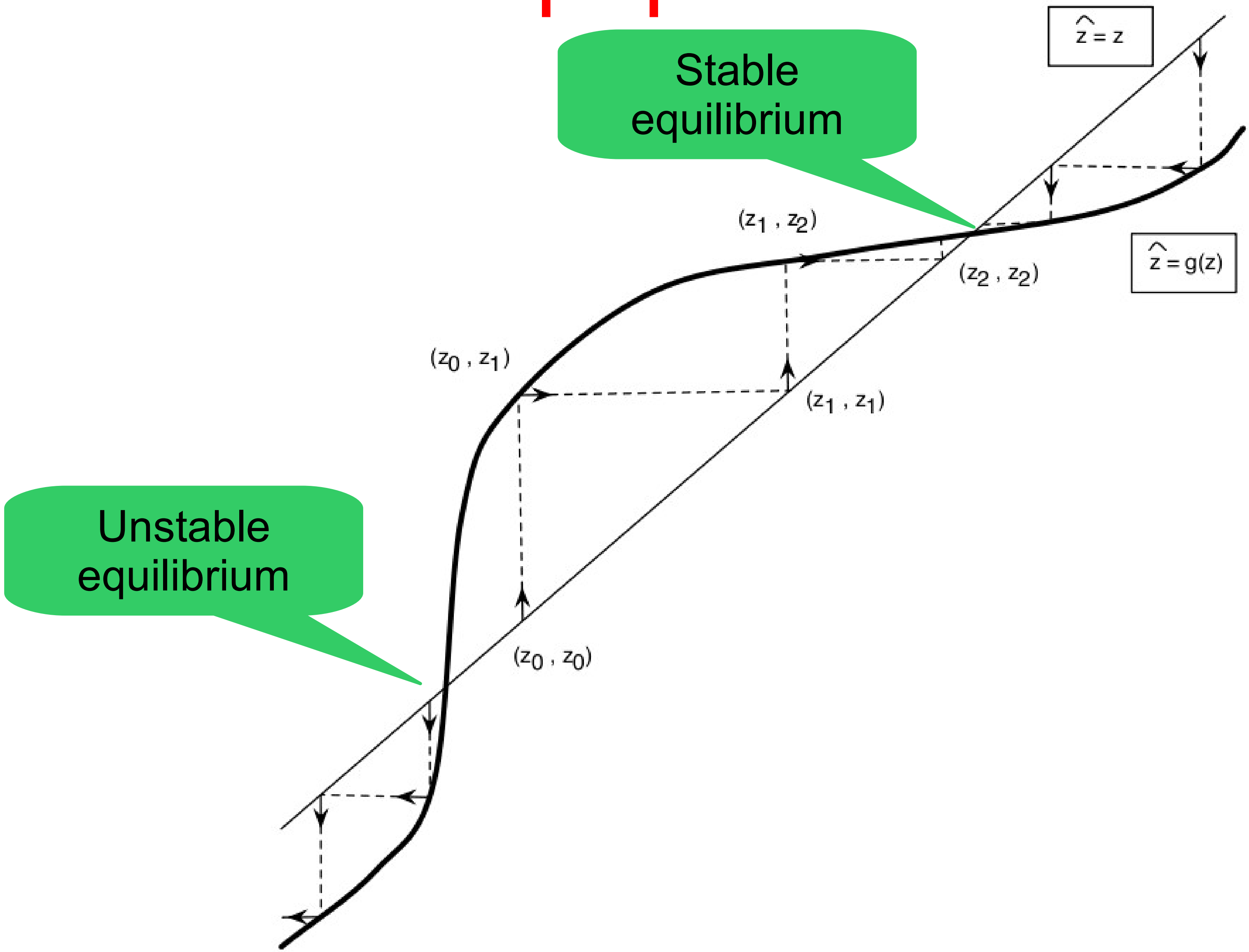
The dynamic behavior of the population

- In the previous slides we considered the simple case of customers buying a (physical) product
- Not, let us apply our knowledge to a different scenario, that is people participating in a large social media site
  - Chat with friends, share videos...
- The number of people participating to the social site of course varies over time
- An initial *audience size* z_0 takes part to the site
  - z_t = audience at time period t, t=0, 1, ...
- At any time, *people evaluate whether to participate based on a shared expectation that the audience size will be the same as what it was in the previous period*

The dynamic behavior of the population

- The function g() maps shared expectations to outcomes
- $z_1 = g(z_0)$
  - Everyone acts in period t=1 on the expectation that the audience size will be z_0
- $z_2 = g(z_1)$
  - In period t=2 everyone will act based on the expectation that the audience size is now z^1
- In general: $z_t = g(z_{t-1})$

The dynamic behavior of the population



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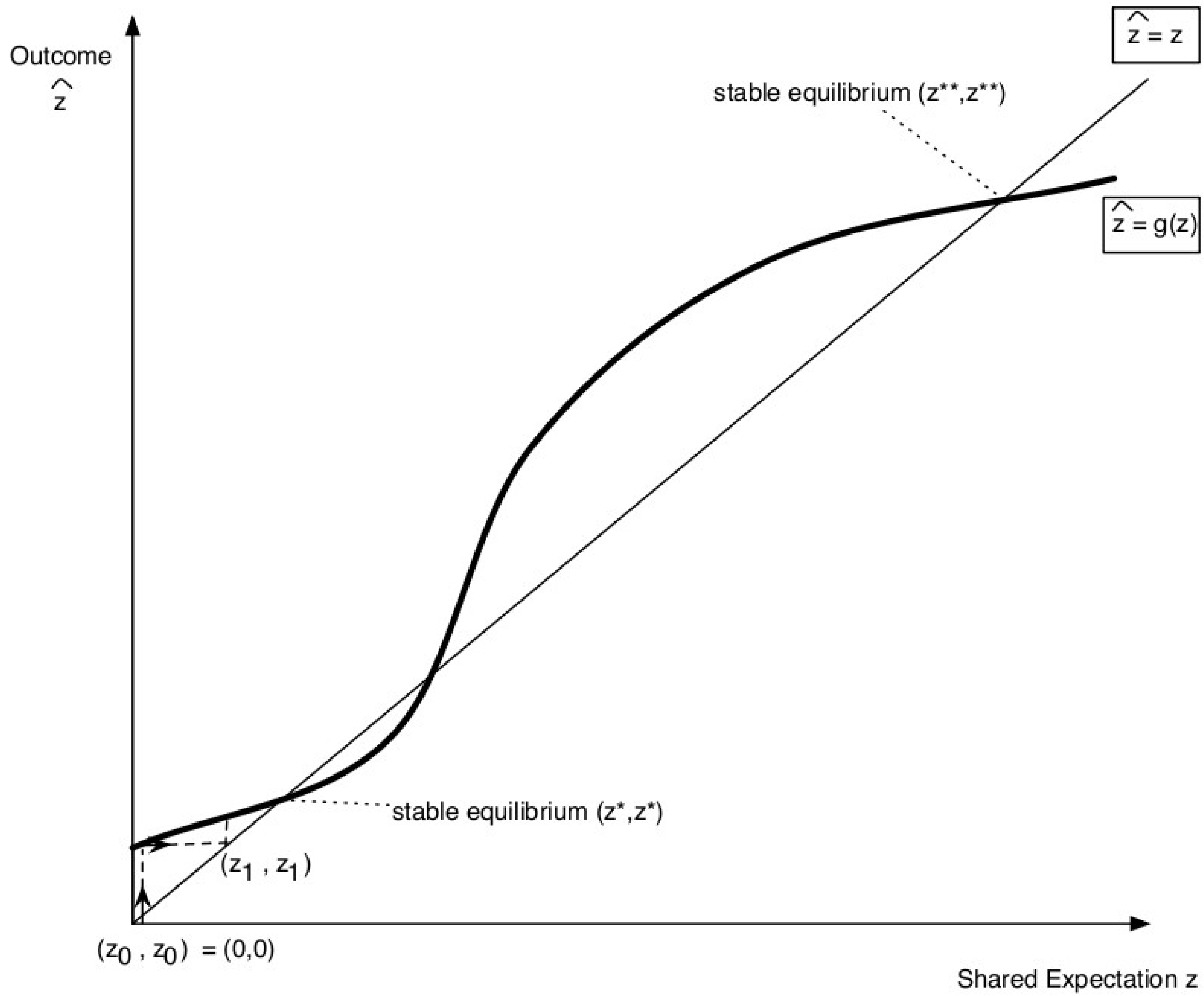
A more complex example

- So far we assumed f(0) = 0
  - Thus, an audience size of zero is a stable equilibrium: if everyone expected that no one would use the product, then no one would
- What does it happen if f(0) > 0 ?
  - In this case the product is considered valuable to people even if they are the only users
  - An audience of zero is no longer an equilibrium

A more complex example



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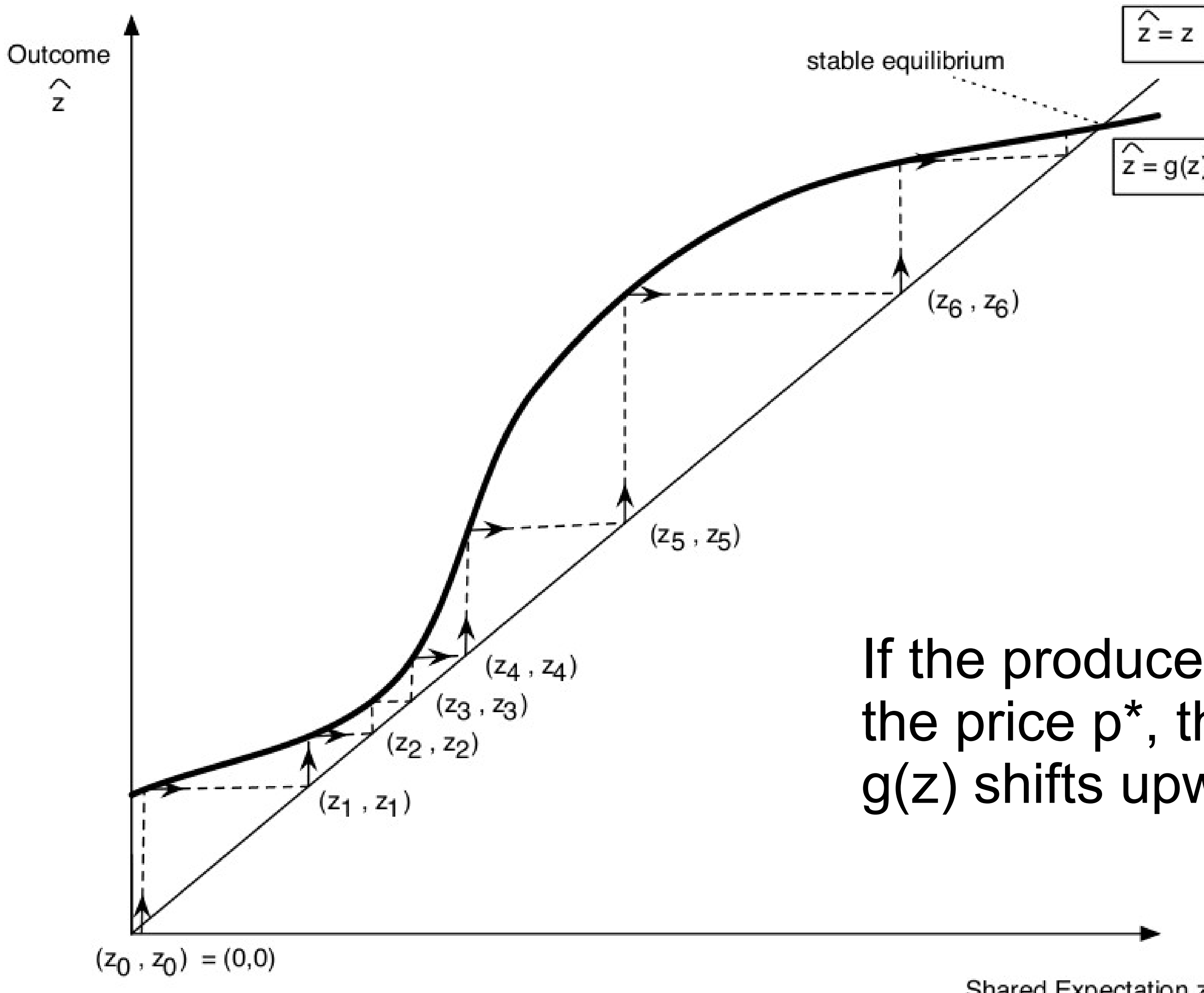
A more complex example



If the producer lowers the price p*, the curve g(z) shifts upwards

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Recap

- If $f(0) = 0$, the firm marketing the product needed alternate ways to get over its tipping point at the low, unstable equilibrium in order to have any customers at all
- When $f(0) > 0$, the audience can grow from zero up to some larger stable equilibrium through the simple dynamics just shown
  - We are able to talk here about an audience that grows gradually and organically, starting from no users at all, rather than one that needs to be pushed by other means over an initial tipping point