The qnetworks Toolbox

A Software Package for Queueing Networks Analysis

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Talk Outline

Talk outline

What is qnetworks?

qnetworks is a package for Queueing Networks analysis.

- Written in GNU Octave, an interpreted language for numerical computations
- qnetworks implements the most important algorithms for QN analysis (MVA, convolution, Jackson networks...);
- qnetworks also provides some numerical algorithms for Markov Chains analysis (computation of steady-state residency probabilities, mean time to absorption, time-averaged sojourn time...);
- Open source software (GNU GPL version 3)

http://www.moreno.marzolla.name/software/qnetworks

Yet Another QN Package?

- Over the years, quite a lot of QN packages have been developed (see `http://web2.uwindsor.ca/math/hlynka/qsoft.html` for a list)
- Unfortunately, most of those packages have vanished; the surviving ones are of very limited scope, obsolete and/or unmaintained

GNU Octave

- GNU Octave is a free Matlab®clone. Octave/Matlab are widely used for implementing numerical algorithms, due to their very compact notation.
- `qnetworks` is entirely written as m-scripts which run inside the Octave interpreter.
- This allows maximum portability, as the Octave interpreter runs on the most common platforms (Windows, MaxOSX, Linux and most UNIX variants).
- The execution speed of the interpreter is not a limiting factor for `qnetworks` (see later).

Talk outline

GNU Octave
Example

Solve $Ax = b^T$ where $A \leftarrow \begin{pmatrix} 12 & 8 & 7 \\ 21 & -9 & 8 \\ 3 & 3 & 1 \end{pmatrix}$ and $b \leftarrow (0.75\ 1\ 3)$.

```
octave> A = [12 8 7; 21 -9 8; 3 3 1];
octave> b = [0.75 1 3];
octave> A\b'
ans =

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   0.56686
  -2.99419
```

GNU Octave
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GNU Octave
Vector manipulation

GNU Octave provides useful features for manipulating vectors

```
octave> a = [1 7 6 1 3 8 2 9 4 2]
a =
   1   7   6   1   3   8   2   9   4   2
octave> sum(a)
ans =  43
octave> a>3
ans =
   0   1   1   0   0   1   0   1   1   0
octave> a(a>3)
ans =
   7   6   8   9   4
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```

Octave
MVA algorithm

MVA algorithm

for $k \leftarrow 1$ to k do
  $Q_k \leftarrow 0$
endfor
for $n \leftarrow 1$ to N do
  for $k \leftarrow 1$ to K do
    if k is a delay center then
      $R_k \leftarrow D_k$
    else
      $R_k \leftarrow D_k(1 + Q_k)$
    endif
  $X \leftarrow n/\left(Z + \sum_{k=1}^{K} R_k\right)$
  for $k \leftarrow 1$ to K do
    $Q_k \leftarrow XR_k$
  endfor
endfor

Octave implementation

```
## dd(k) == 1 iff server k is delay center
function [R,Q] = mva(N, V, D, Z, dd)
  Q = zeros(size(V));
  for n=1:N
    R = D .* (1 + Q.*(dd == 0));
    X = n / (Z+sum(R));
    Q = X*R;
  endfor
endfunction
```

Octave
MVA algorithm

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```

Note on notation

What does the expression `D .* (1 + Q .* (dd == 0))` compute?

Let:

```
D = [15 3 6];
Q = [1 7.4 2];
dd = [1 0 0];
```

- is the boolean vector [0 1 1]
- is the element by element product of [1 7.4 2] and [0 1 1] ($\rightarrow$ [0 7.4 2])
- adds 1 to all elements of [0 7.4 2] ($\rightarrow$ [1 8.4 3])
- is the element by element product of [15 3 6] and [1 8.4 3] ($\rightarrow$ [15 25.2 18])

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`dd==0`

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Q = [1 7.4 2];
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```

```
1 + Q.*(dd == 0)
```

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Note on notation

Thus, the expression

```
R = D .* (1 + Q.*(dd == 0))
```

is equivalent to this pseudo code:

for $k \leftarrow 1$ to K do
  if k is a delay center then
    $R_k \leftarrow D_k$
  else
    $R_k \leftarrow D_k(1 + Q_k)$
  endif
endfor

Talk outline

The qnetworks Toolbox

Single-station Queueing Systems

qnetworks computes utilization, response time, average number of requests and throughput of several kinds of single-station queueing systems:

- $M/M/1$, $M/M/m$
- $M/M/1/k$, $M/M/m/k$
- $M/M/\infty$
- asymmetric $M/M/m$ which contains m servers with possibly different service rates
- $M/G/1$ (general service time distribution)
- $M/H_m/1$ (hyperexponential service time distribution)

Example
Single Station Queueing Systems

Let us consider a $M/M/1$ system

- Arrival rate $\lambda = 0.3$ jobs/second
- Service rate $\mu = 0.4$ jobs/sec

We can compute the utilization U, response time R, average number of requests in the system Q and throughput X as follows:

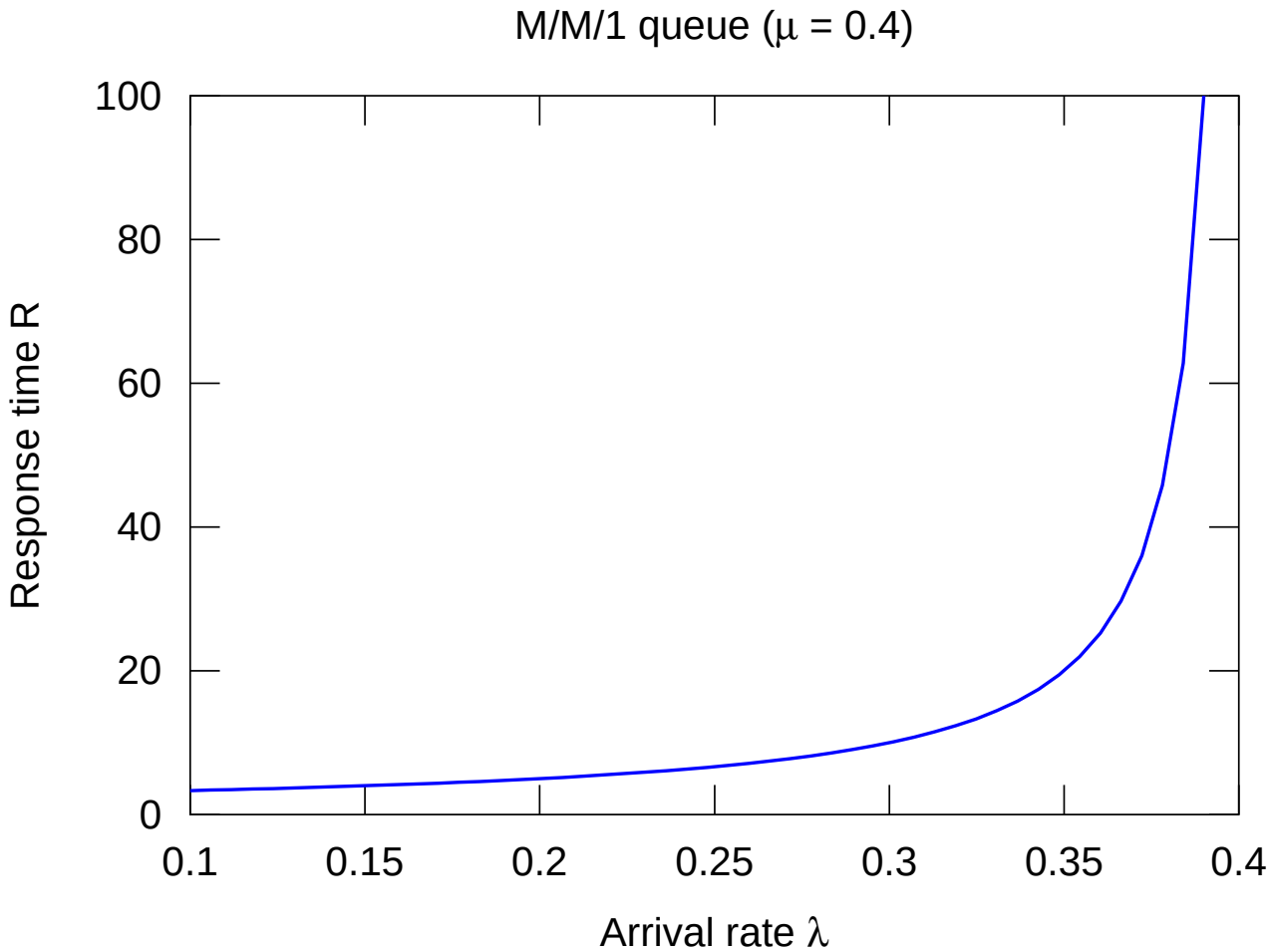
```
octave> lambda = 0.3;
octave> mu = 0.4;
octave> [U R Q X] = qnmm1(lambda, mu)
U =  0.75000
R = 10.0000
Q =  3.0000
X =  0.30000
```

Example
Single Station Queueing Systems

Let us examine how the response time R grows as the arrival rate λ approaches the service rate $\mu = 0.4$

```
mu = 0.4;
lambda = linspace(0.1,0.39,50);
R = zeros(size(lambda));
for i=1:length(lambda)
  [nc R(i)]=qnmm1(lambda(i),mu);
endfor
plot(lambda,R);
```

Example
Single Station Queueing Systems

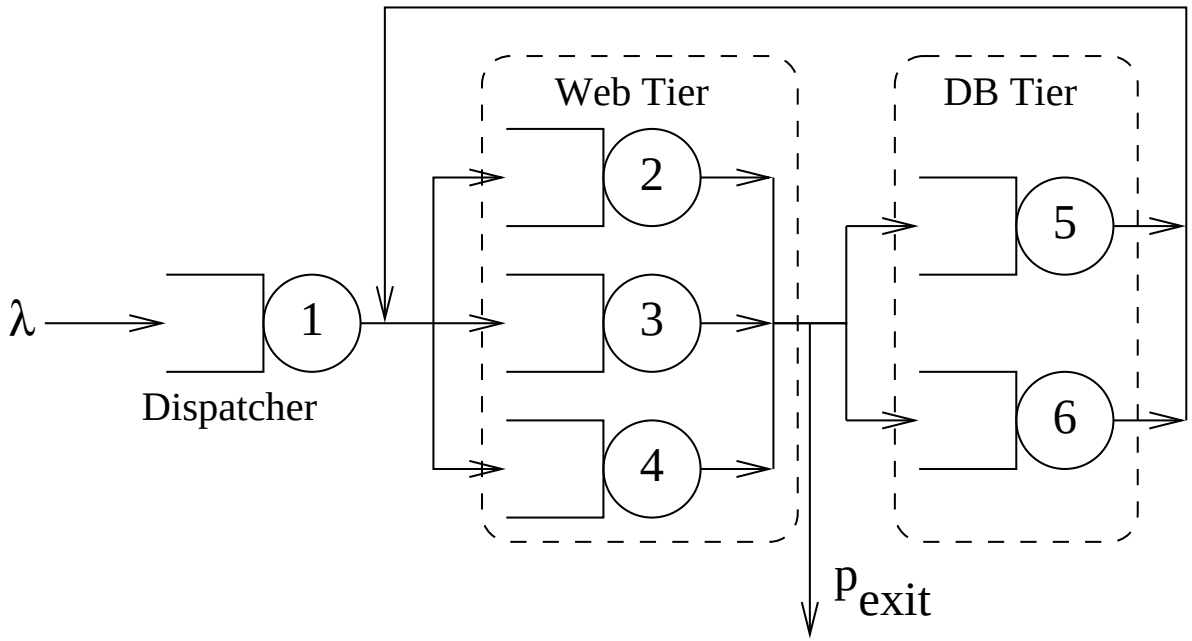


Queueing Networks

Function Name	Supported network type			
	Open	Closed	Single	Multi
`qnopensingle()`	√	–	√	–
`qnopenmulti()`	√	–	–	√
`qnconvolution()`	–	√	√	–
`qnconvolutionld()`	–	√	√	–
`qnclosedsinglemva()`	–	√	√	–
`qnclosedsinglemvald()`	–	√	√	–
`qnclosedmultimva()`	–	√	–	√
`qnclosedmultimvaapprox()`	–	√	–	√
`qnmix()`	√	√	–	√
`qnsolve()`	√	√	√	√
`qnmvablo()`	–	√	√	–
`qnmarkov()`	–	√	√	–
`qnopenab()`	√	–	√	–
`qnclosedab()`	–	√	√	–
`qnopenbsb()`	√	–	√	–
`qnclosedbsb()`	–	√	√	–
`qnclosedgb()`	–	√	√	–

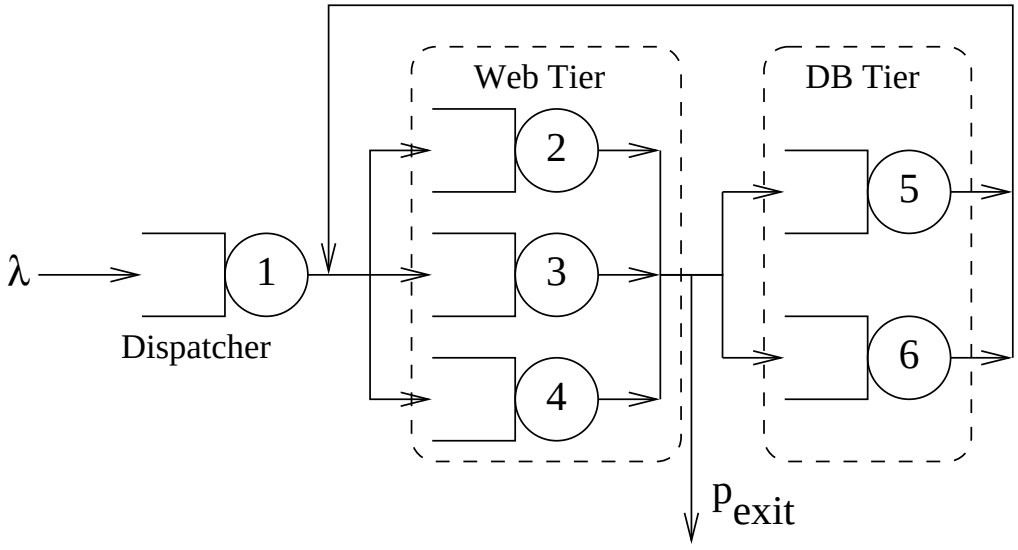
Queueing Networks

We illustrate `qnetworks` capabilities for solving QNs using a simple example. We consider a three-tier E-commerce site



- Dispatcher (center 1, service time $S_1 = 0.5$);
- Web Servers (centers 2–4, service time $S_2 = S_3 = S_4 = 0.8$);
- Database Servers (centers 5–6, service time $S_5 = S_6 = 1.8$);

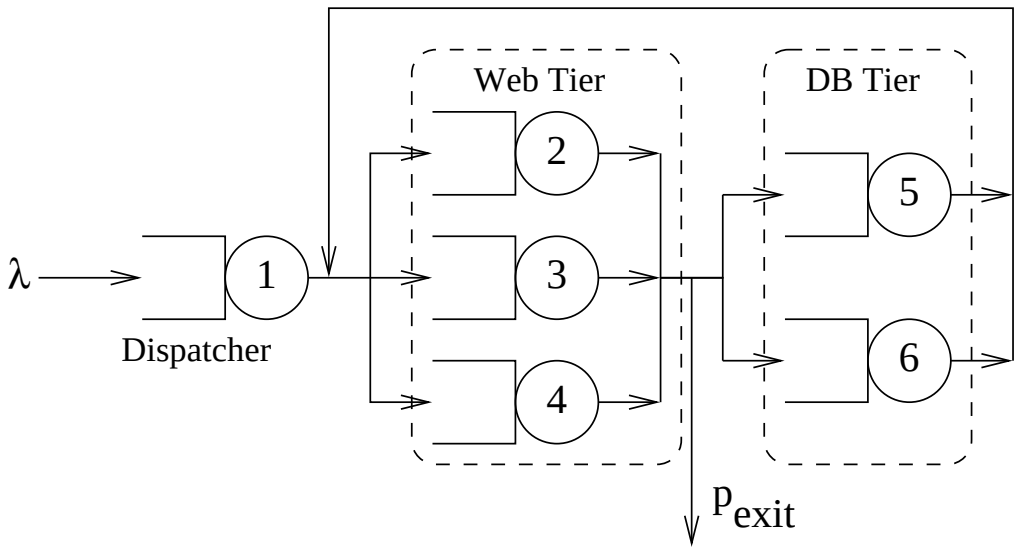
E-Commerce example
Routing probability matrix



Assuming $p_{\text{exit}} = 0.5$ and uniform routing, the routing probability matrix P is

$$P = \begin{pmatrix} 0 & 1/3 & 1/3 & 1/3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/4 & 1/4 \\ 0 & 0 & 0 & 0 & 1/4 & 1/4 \\ 0 & 0 & 0 & 0 & 1/4 & 1/4 \\ 0 & 1/3 & 1/3 & 1/3 & 0 & 0 \\ 0 & 1/3 & 1/3 & 1/3 & 0 & 0 \end{pmatrix}$$

E-Commerce example
Building the model




```
p_exit = 0.5; # exit probability
i = 2:4; # indexes of Web servers
j = 5:6; # indexes of DB servers
P = zeros(6,6);
P(1,i) = 1/3;
P(i,j) = (1-p_exit)/2;
P(j,i) = 1/3;
S = [0.5 0.8 0.8 0.8 1.8 1.8];
lambda = [0.1 0 0 0 0 0];
V = qnvisits(P,lambda);

[U R Q X]= \
    qnopensingle(sum(lambda),S,V);
```

E-commerce example
Results

The statement `V = qnvisits(P,lambda)` computes the visit counts V_k, which satisfy:

$$V_k = \lambda_k + \sum_{j=1}^{K} V_j P_{jk}$$

In the previous example we have

$$V_1 = 1,\ V_2 = V_3 = V_4 = 0.6\bar{6} \text{ and } V_5 = V_6 = 0.5$$

and we obtain the following performance results:

$$\begin{aligned}
U &= (0.050000,\ 0.053333,\ 0.053333,\ 0.053333,\ 0.090000,\ 0.090000) \\
R &= (0.526320,\ 0.845070,\ 0.845070,\ 0.845070,\ 1.978020,\ 1.978020) \\
Q &= (0.052632,\ 0.056338,\ 0.056338,\ 0.056338,\ 0.098901,\ 0.098901) \\
X &= (0.100000,\ 0.066667,\ 0.066667,\ 0.066667,\ 0.050000,\ 0.050000)
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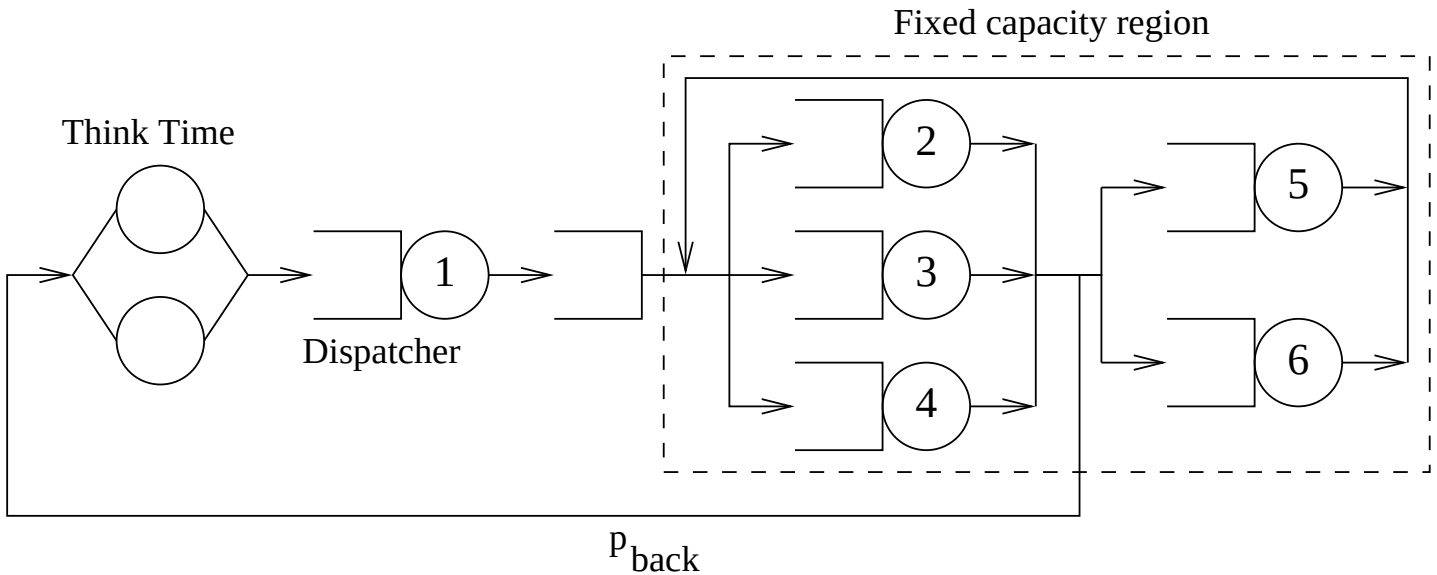
E-commerce example
Flow-Equivalent Centers

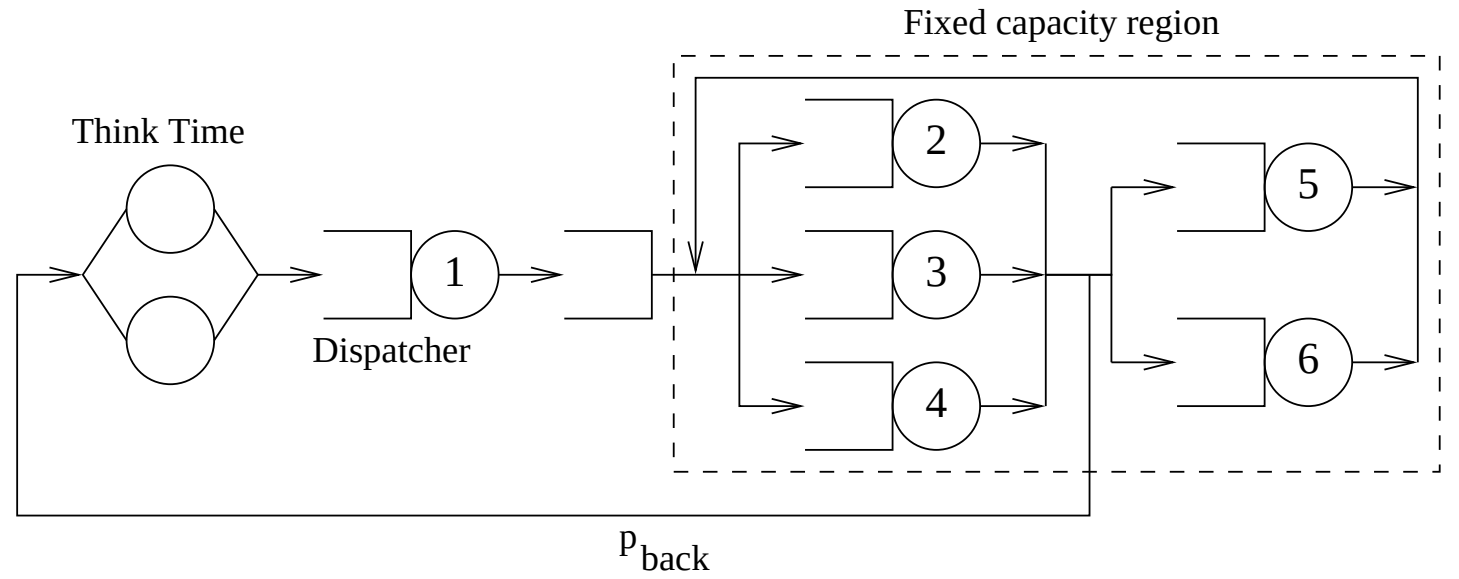
Let we consider a slightly more sophisticated example.

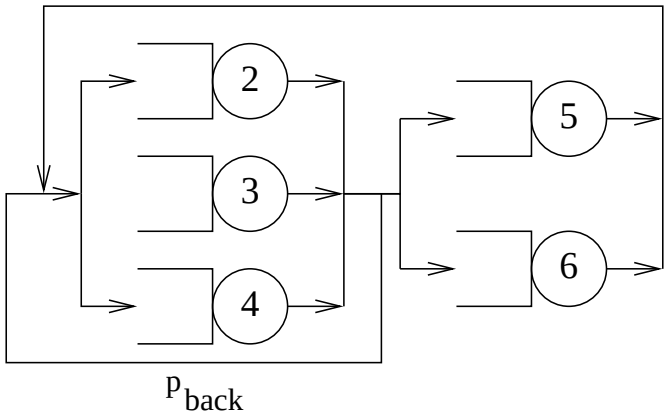
- The system is modeled as a closed network with N customers.
- There is a capacity constraint of M requests behind the dispatcher

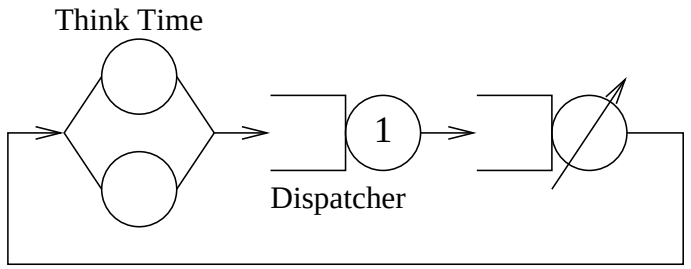


E-commerce example
Flow-Equivalent Centers

We replace the fixed capacity region with a load-dependent service center and solve the resulting model (which does have product form solution).







Analyzing the capacity-constrained model

We start by defining the complete model

```
p_back = 0.5;     # back probability
i = 2:4;          # range of Web servers
j = 5:6;          # range of DB servers
P = zeros(6,6);
P(1,i) = 1/3;
P(i,j) = (1-p_back)/2;
P(i,1) = p_back;
P(j,i) = 1/3;
S = [0.5 0.8 0.8 0.8 1.8 1.8];
V = qnvisits(P); # Compute visit counts
Z = 5;           # Think Time
N = 20;          # Population
m = ones(1,6);   # m(k)=number of servers at center k
```

Then we short-circuit the capacity constrained region by setting all other service times to zero

```
S(1) = 0
```

Analyzing the capacity-constrained model

We solve the short-circuited submodel by computing the throughput $X_{\text{sub}}(n)$ along the removed node(s) as a function of the population size $n = 1, 2, \ldots M$:

$$S_{\text{sub}}(n) = \begin{cases} 1/X_{\text{sub}}(n) & \text{if } 1 \leq n \leq M \\ 1/X_{\text{sub}}(M) & \text{if } M < n \leq N \end{cases}$$

This can be done with the following code:

```
Ssub = zeros(1,N); # Initialize to zero
M = 10;            # Capacity constraint
for n=1:M
  [U R Q X] = qnclosedsinglemva(n,S,V);
  Ssub(n) = V(1)/X(1);
endfor
Ssub(M+1:N) = Ssub(M);
```

Analyzing the capacity-constrained model

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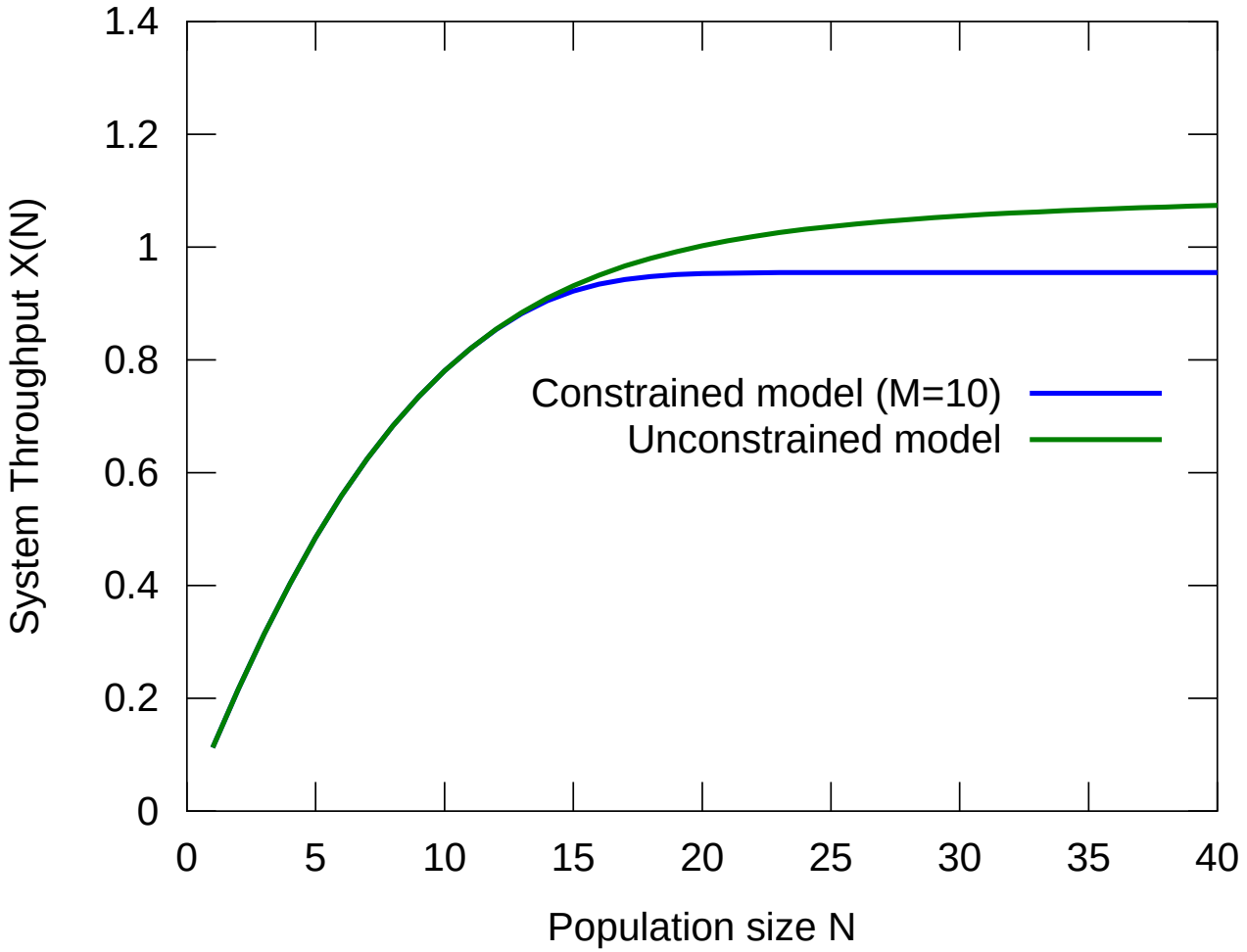
  [U R Q X] = qnclosedsinglemva(n,S,V);
  Ssub(n) = V(1)/X(1);
endfor
Ssub(M+1:N) = Ssub(M);
```

Analyzing the capacity-constrained model

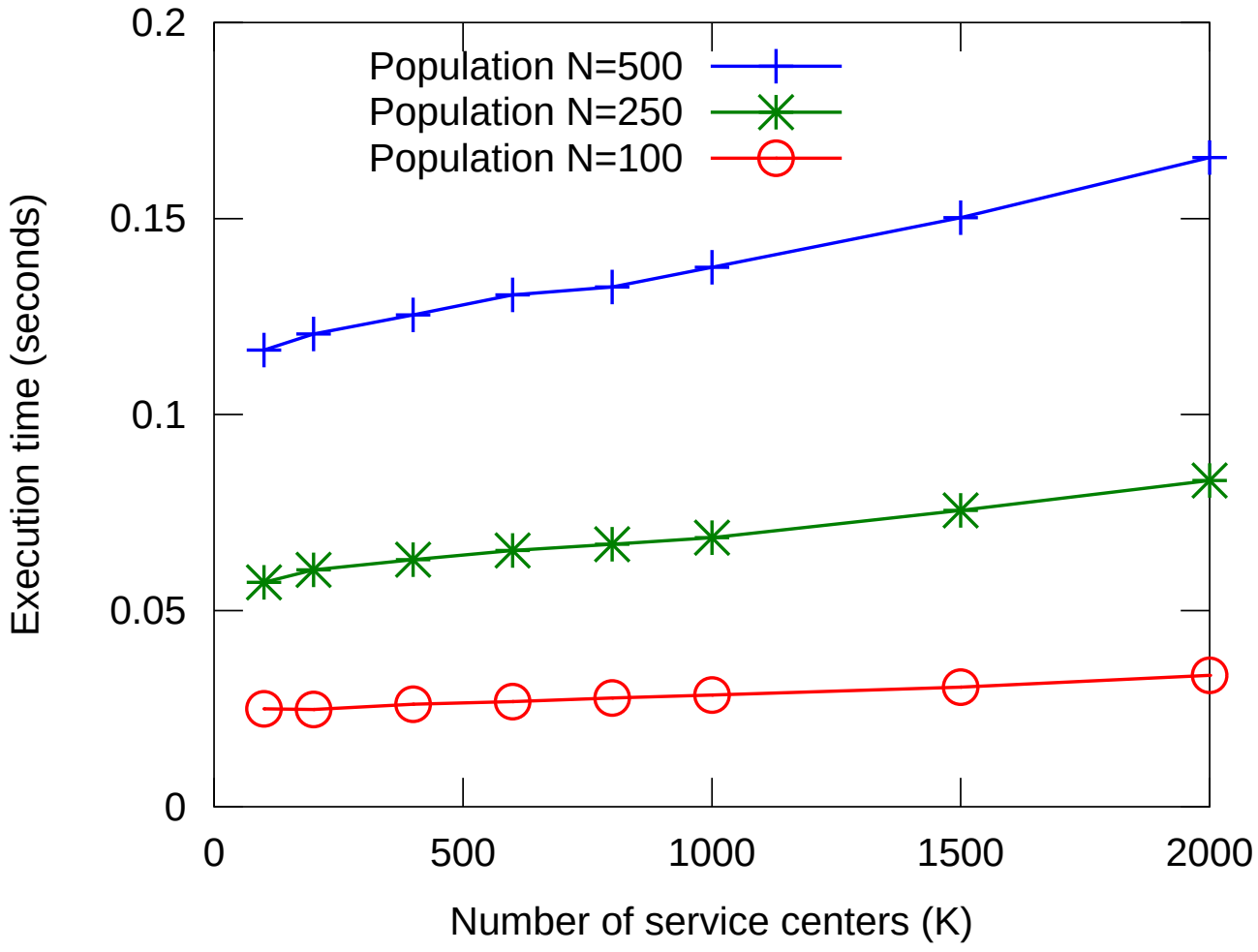
We can now solve the equivalent model)with the capacity constrained region replaced by the FESC)

```
S = [ 0.5*ones(1,N); Ssub ];
V = [1 1];
Z = 5;
[U R Q X] = qnclosedsinglemvald(N,S,V,Z);
```

Analyzing the capacity-constrained model



Performance Considerations

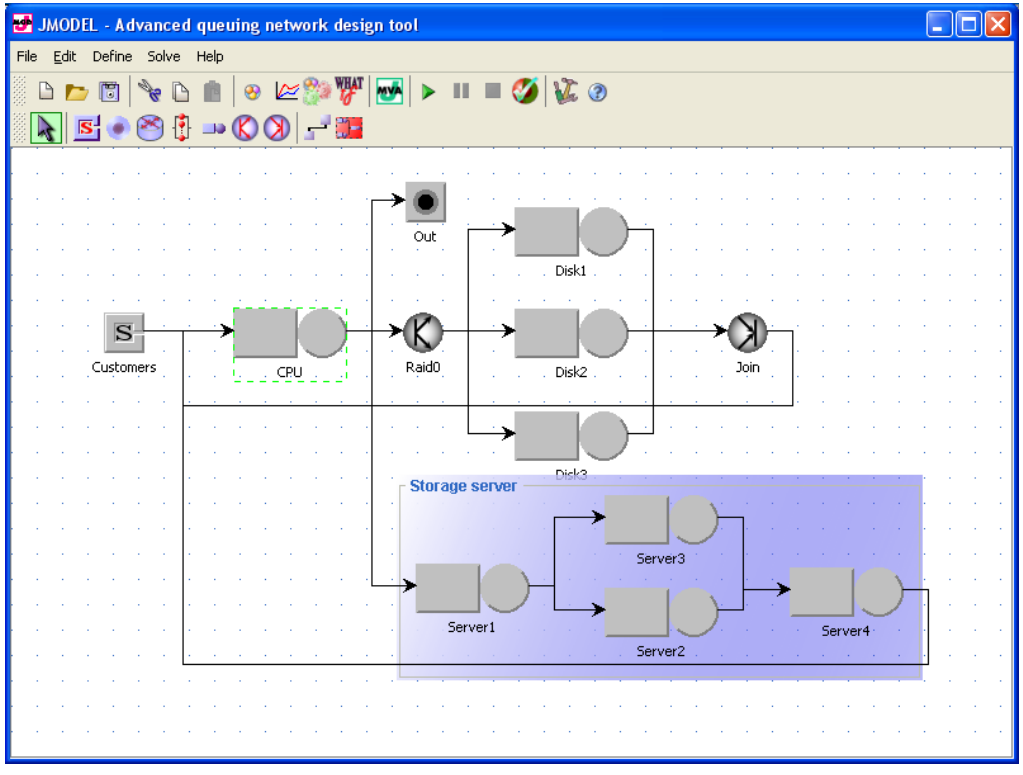


Execution times of the `qnclosedsinglemva` function on a Linux PC, AMD Athlon 64 X2 Dual Core processor 3800+ with 2GB of RAM. Each data point is the average execution time of 5 runs.

Talk outline

Java Modelling Tools (JMT)

JMT is a nice alternative to `qnetworks`. JMT is written in Java and is available at `http://jmt.sourceforge.net/`



JMT vs qnetworks

	JMT	qnetworks
Solution technique	Simulation	Analytical
Graphical user interface	Yes	No
Allows to programmatically build and solve models?	No	Yes
Supports Product-Form networks?	Yes	Yes
Supports Non PF networks?	Yes	Some
Free and open source?	Yes (GPLv2)	Yes (GPLv3)

Talk outline

Conclusions

`qnetworks` is an open source QN analysis package for GNU Octave. It supports:

- single station queueing systems;
- open, closed or mixed networks with multiple job classes;
- the MVA (single and multi class) and convolution (single class only) algorithm for product-form networks;
- computation of performance bounds (Asymptotic Bounds, Balanced System Bounds, Geometric Bounds);
- approximate MVA for closed QN with blocking;
- Markov Chains analysis

Many new algorithms still to be implemented. Would you like to help?

`http://www.moreno.marzolla.name/software/qnetworks/`

Talk outline

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