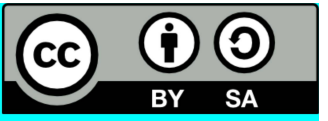
# Parallelizing Loops

Moreno Marzolla
Dip. di Informatica—Scienza e Ingegneria (DISI)
Università di Bologna

http://www.moreno.marzolla.name/



# Credits

- Salvatore Orlando (Univ. Ca' Foscari di Venezia)
- Mary Hall (Univ. of Utah)

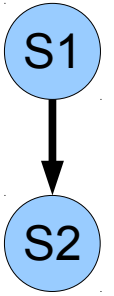
# Loop optimization

- 90% of execution time in 10% of the code
  - Mostly in loops
- Loop optimizations
  - Transform loops preserving the semantics
- Goal
  - Single-threaded system: mostly optimizing for memory hierarchy
  - Multi-threaded and vector systems: loop parallelization

# Data Dependence

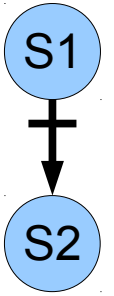
- Two memory accesses are involved in a data dependence if they may refer to the same memory location and one of the references is a write
- Data-Flow or true dependence
  - RAW (Read After Write)

```
S1  a = b + c;
S2  d = 2 * a;
```



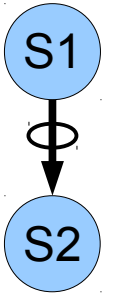

- Anti dependence
  - WAR (Write After Read)

```
S1  c = a + b;
S2  a = 2 * a;
```




- Output dependence
  - WAW (Write After Write)

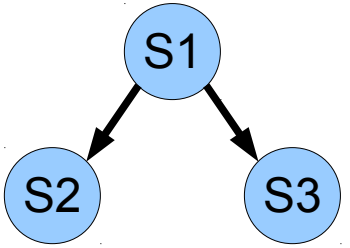
```
S1  a = k;
    if (a>0) {
S2      a = 2 * c;
    }
```



# Control Dependence

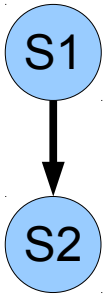
- An instruction S2 has a control dependence on S1 if the outcome of S1 determines whether S2 is executed or not
  - Of course, S1 and S2 can not be exchanged
- This type of dependence applies to the condition of an if-then-else or loop with respect to their bodies

```
S1  if (a>0) {
S2        a = 2 * c;
    } else {
S3        b = 3;
    }
```



# Dependence

- In the following, we always use a simple arrow to denote any type of dependence
  - S2 depends on S1

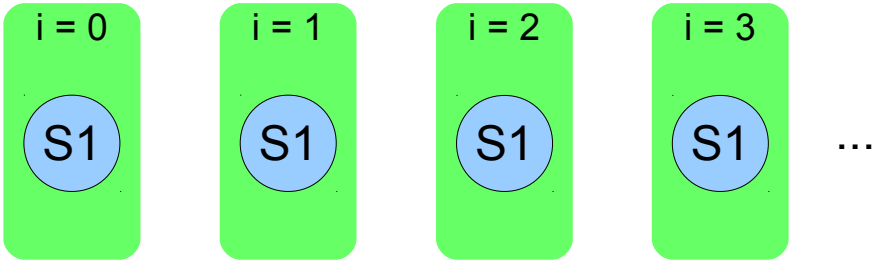


# Fundamental Theorem of Dependence

- Any reordering transformation that preserves every dependence in a program preserves the meaning of that program
- Recognizing parallel loops (intuitively)
  - Find data dependences in loop
  - No dependences crossing iteration boundary → parallelization of loop's iterations is safe

# Example

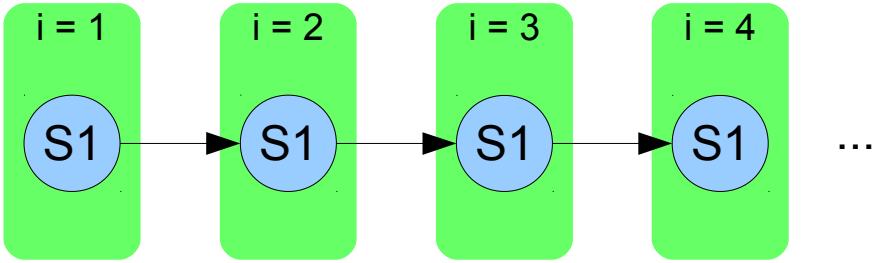
```
for (i=0; i<n; i++) {
 S1 a[i] = b[i] + c[i];
}
```




- Each iteration does not depend on previous ones
  - There are no dependences crossing iteration boundaries
- This loop is fully parallelizable
  - Loop iterations can be performed concurrently, in any order
  - Iterations can be split across processors

# Example

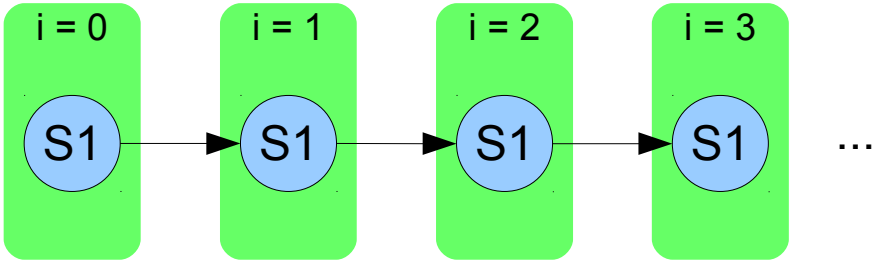
```
for (i=1; i<n; i++) {
 S1 a[i] = a[i-1] + b[i]
}
```




- Each iteration depends on the previous one (RAW)
  - Loop-carried dependence
- Hence, this loop is not parallelizable with trivial transformations

# Example

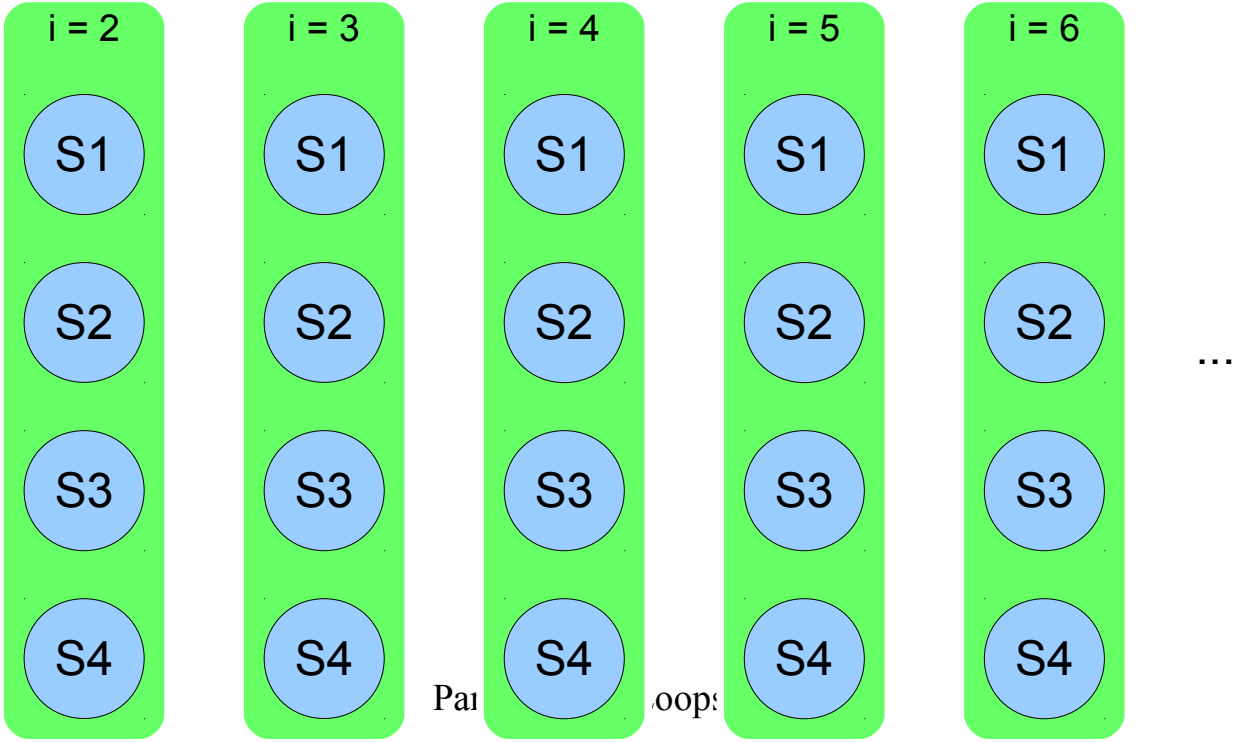
```
s = 0;
for (i=0; i<n; i++) {
S1  s = s + a[i];
}
```




- We have a loop-carried dependency on s that can not be removed with *trivial* loop transformations
  - but can be removed with *non-trivial* transformations
  - this is a reduction, indeed!

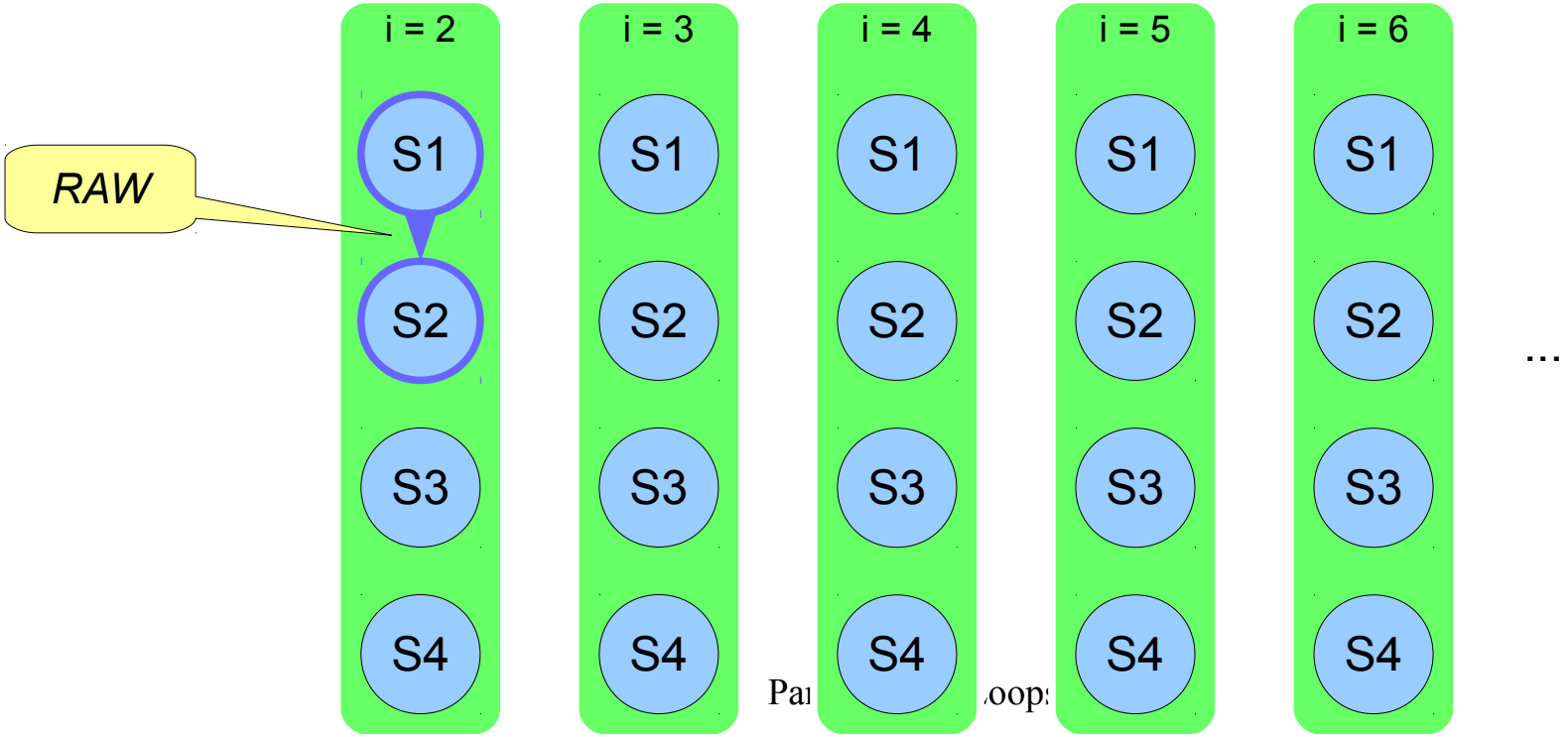
# Exercise: draw the dependencies among iterations of the following loop

```
for (i=2; i<n; i++) {
S1  a[i] = 4 * c[i-1] - 2;
S2  b[i] = a[i] * 2;
S3  c[i] = a[i-1] + 3;
S4  d[i] = b[i] + c[i-2];
}
```



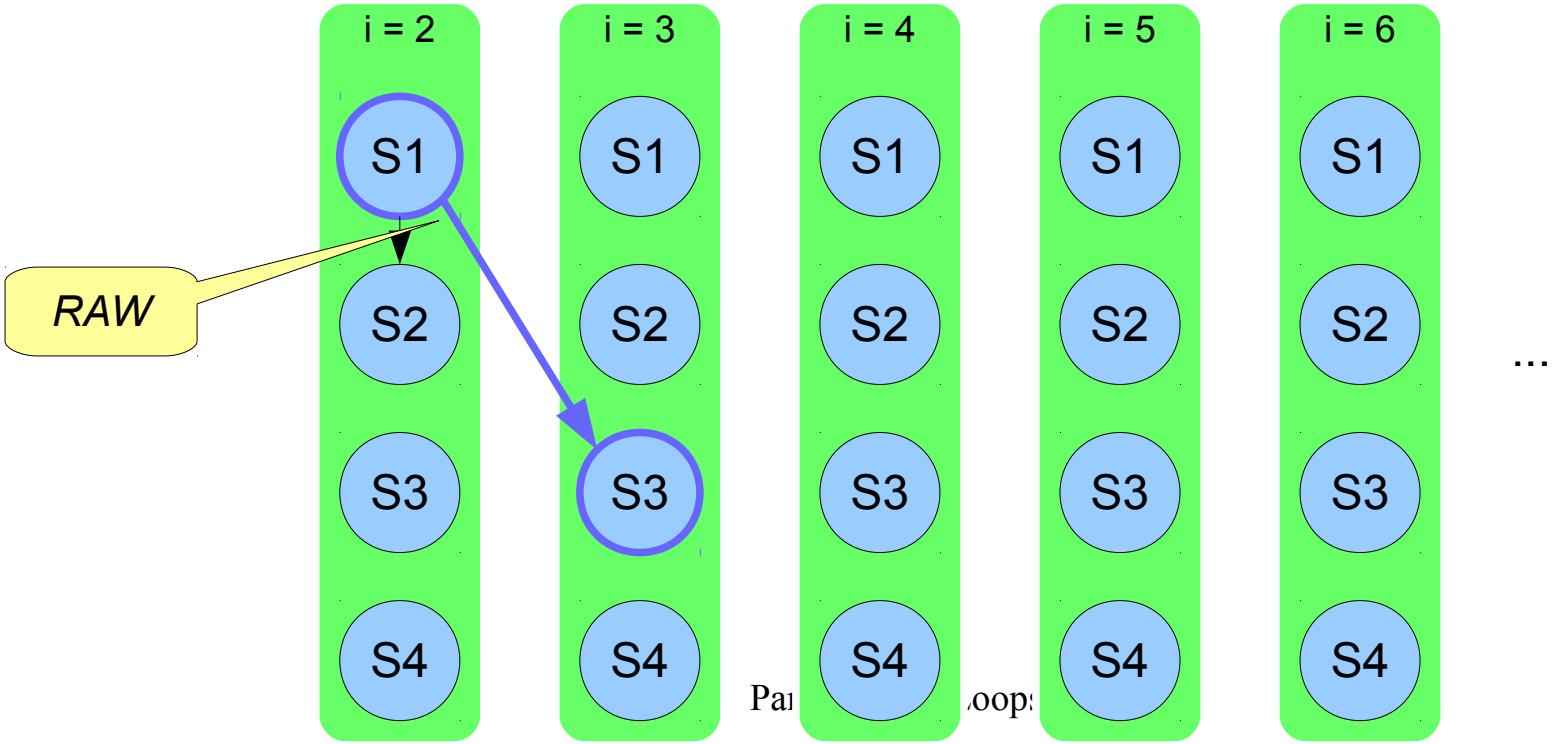
# Exercise (cont.)

```
for (i=2; i<n; i++) {
S1  a[i] = 4 * c[i-1] - 2;
S2  b[i] = a[i] * 2;
S3  c[i] = a[i-1] + 3;
S4  d[i] = b[i] + c[i-2];
}
```



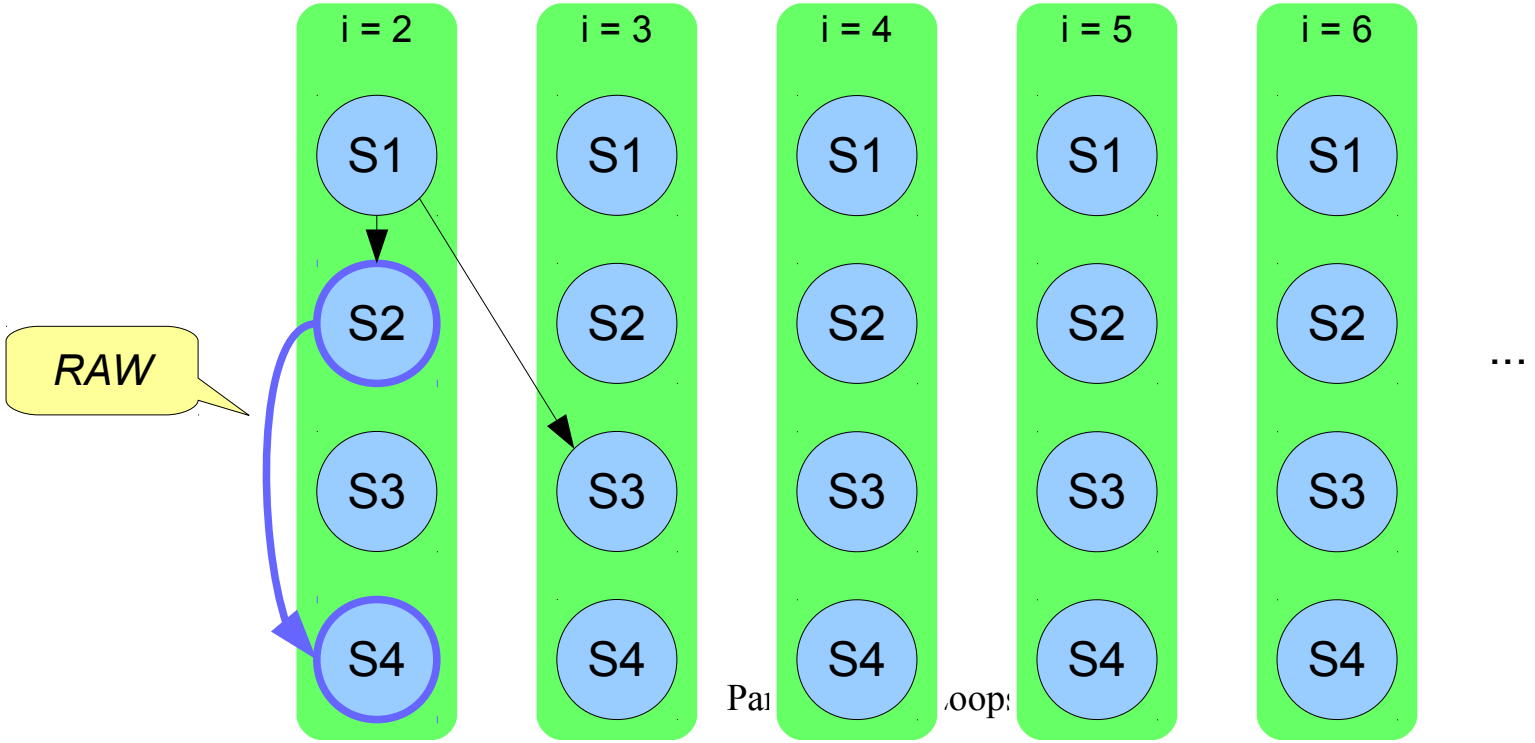
# Exercise (cont.)

```
for (i=2; i<n; i++) {
S1  a[i] = 4 * c[i-1] - 2;
S2  b[i] = a[i] * 2;
S3  c[i] = a[i-1] + 3;
S4  d[i] = b[i] + c[i-2];
}
```



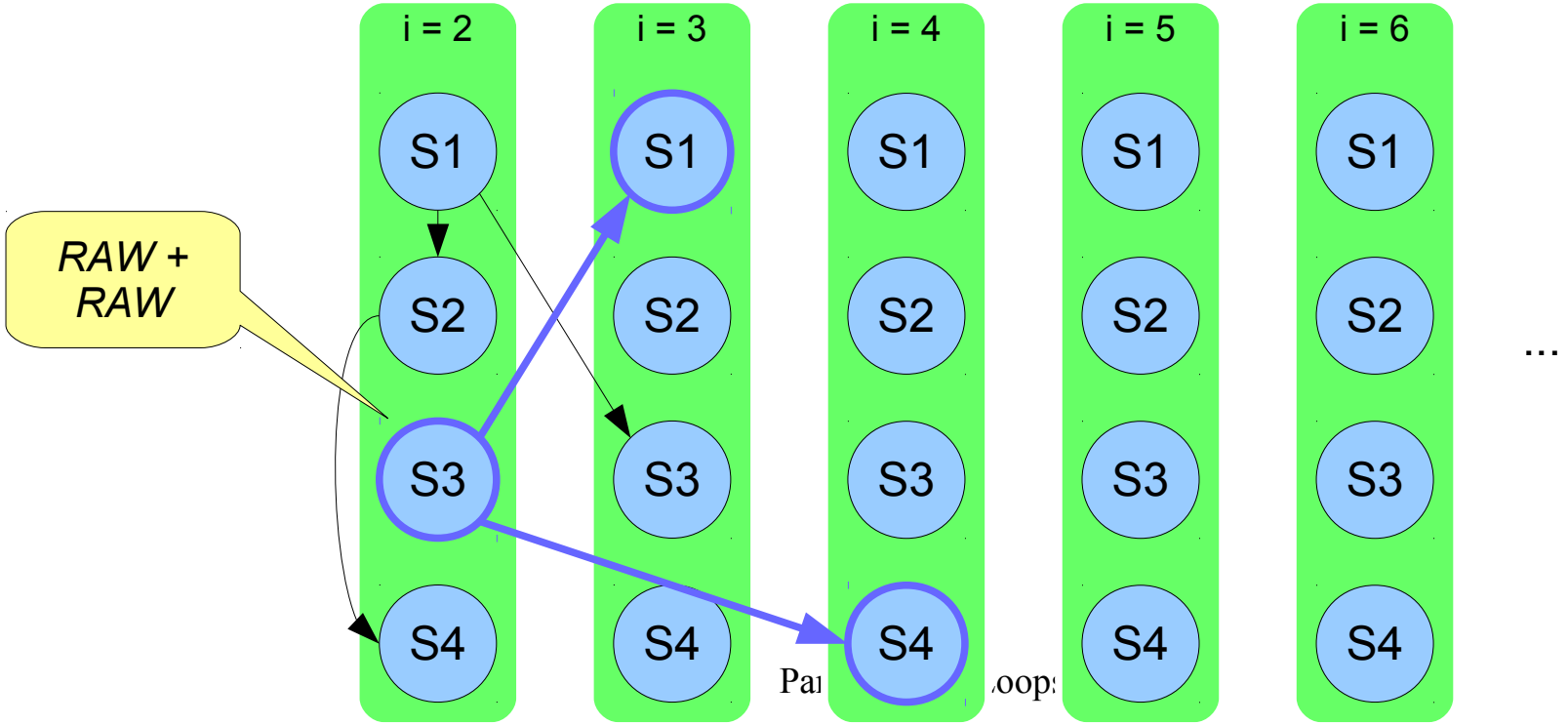
# Exercise (cont.)

```
for (i=2; i<n; i++) {
S1  a[i] = 4 * c[i-1] - 2;
S2  b[i] = a[i] * 2;
S3  c[i] = a[i-1] + 3;
S4  d[i] = b[i] + c[i-2];
}
```



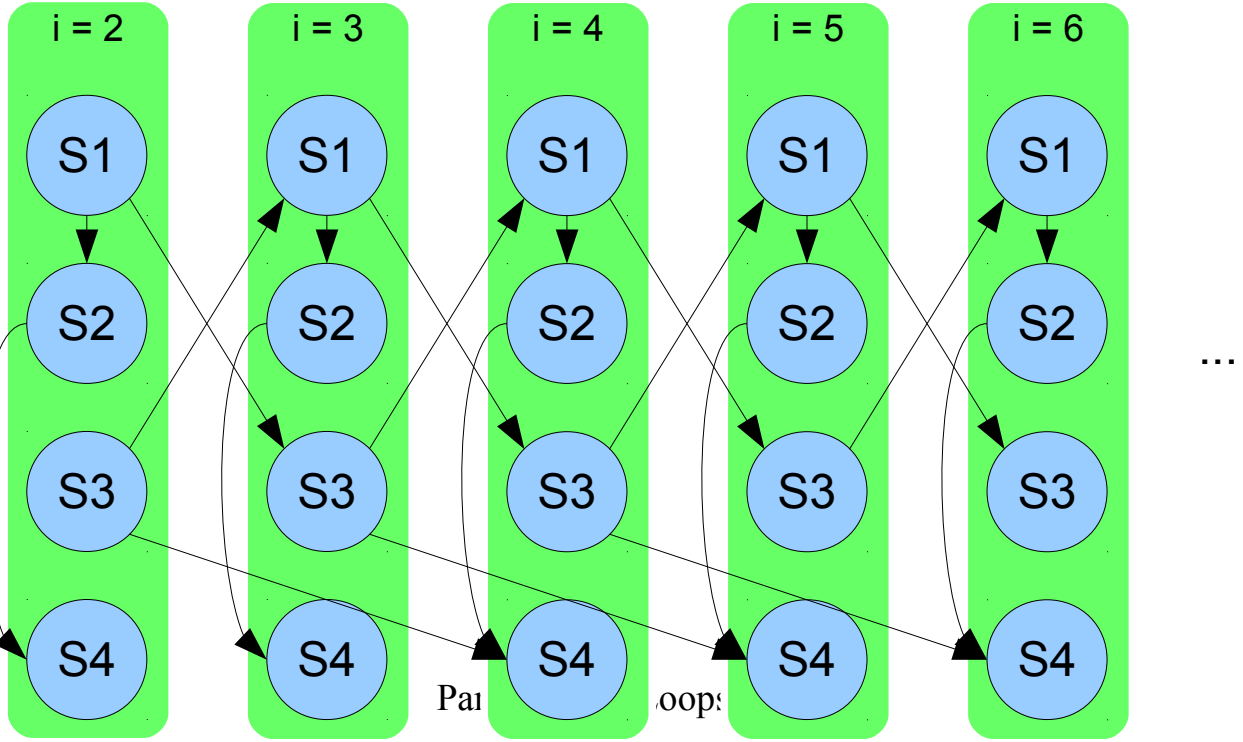
# Exercise (cont.)

```
for (i=2; i<n; i++) {
S1  a[i] = 4 * c[i-1] - 2;
S2  b[i] = a[i] * 2;
S3  c[i] = a[i-1] + 3;
S4  d[i] = b[i] + c[i-2];
}
```



# Exercise (cont.)

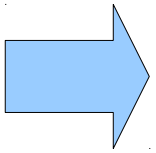
```
for (i=2; i<n; i++) {
S1  a[i] = 4 * c[i-1] - 2;
S2  b[i] = a[i] * 2;
S3  c[i] = a[i-1] + 3;
S4  d[i] = b[i] + c[i-2];
}
```



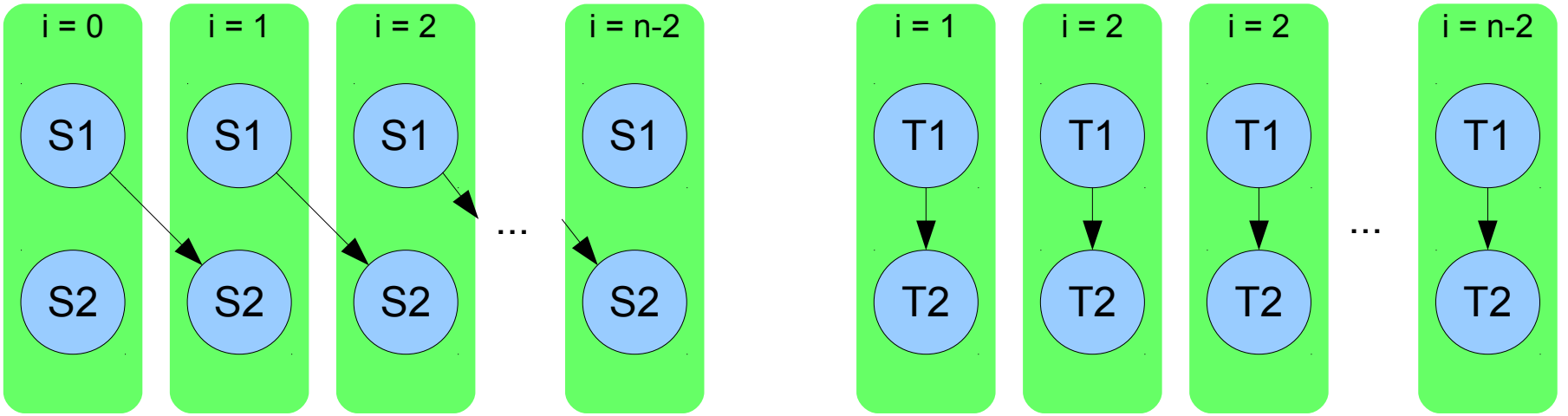
# Removing dependences: Loop aligning

- Dependencies can sometimes be removed by aligning loop iterations

```
a[0] = 0;
for (i=0; i<n-1; i++) {
S1  a[i+1] = b[i] * c[i];
S2  d[i] = a[i] + 2;
}
```



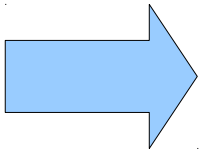
```
a[0] = 0;
d[0] = a[0] + 2;
for (i=1; i<n-1; i++) {
T1  a[i] = b[i-1] * c[i-1];
T2  d[i] = a[i] + 2;
}
a[n-1] = b[n-2] * c[n-2];
```



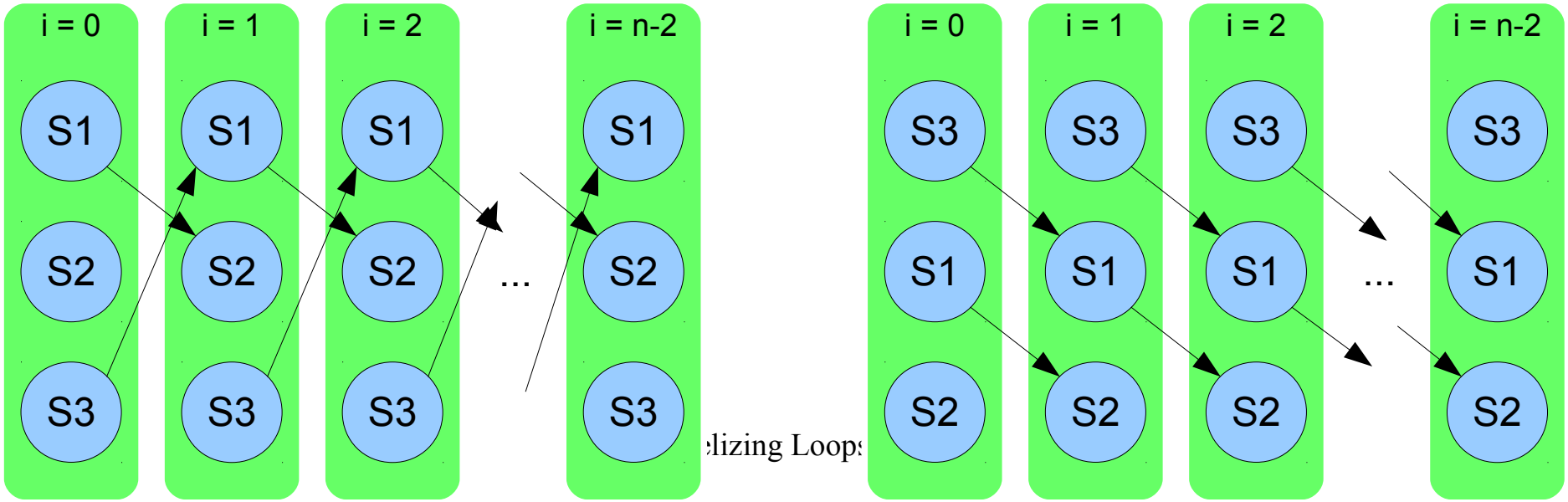
# Removing dependences: Reordering / 1

- Some dependencies can be removed by reordering

```
for (i=0; i<n-1; i++) {
S1  a[i+1] = c[i] + k;
S2  b[i]   = b[i] + a[i];
S3  c[i+1] = d[i] + w;
}
```



```
for (i=0; i<n-1; i++) {
S3  c[i+1] = d[i] + w;
S1  a[i+1] = c[i] + k;
S2  b[i]   = b[i] + a[i];
}
```

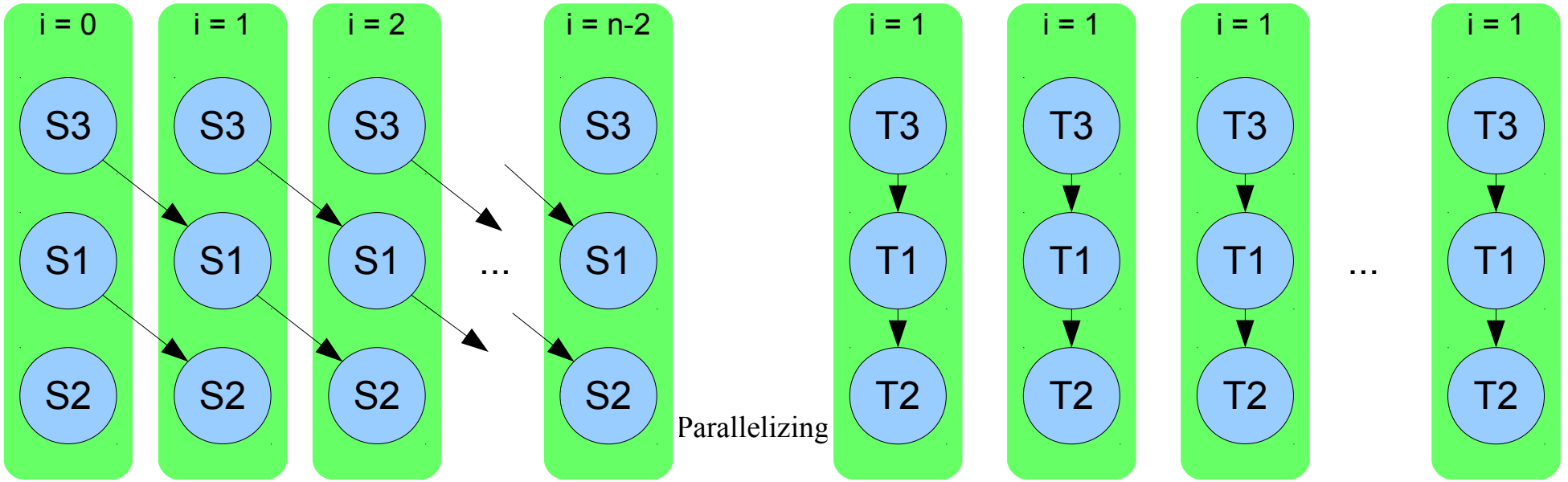


# Reordering / 2

- After reordering, we can align loop iterations

```
for (i=0; i<n-1; i++) {
S3  c[i+1] = d[i] + w;
S1  a[i+1] = c[i] + k;
S2  b[i]   = b[i] + a[i];
}
```

```
b[0] = b[0] + a[0];
a[1] = c[0] + k;
b[1] = c[1] + a[1];
for (i=1; i<n-2; i++) {
T3  c[i] = d[i-1] + w;
T1  a[i+1] = c[i] + k;
T2  b[i+1] = b[i+1] + a[i+1];
}
c[n-2] = d[n-3] + w;
a[n-1] = c[n-2] + k;
c[n-1] = d[n-2] + w;
```



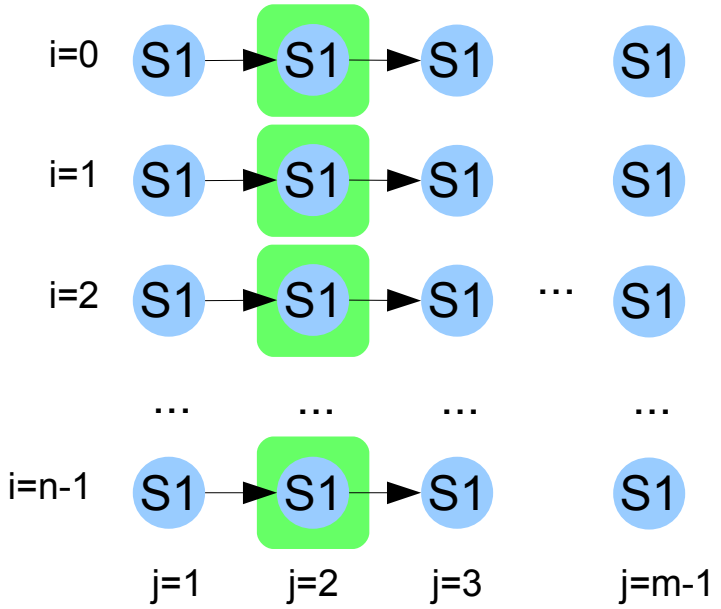
# Loop interchange

- Exchanging the loop indexes might allow the outer loop to be parallelized
  - Why? To use coarse-grained parallelism (if appropriate)

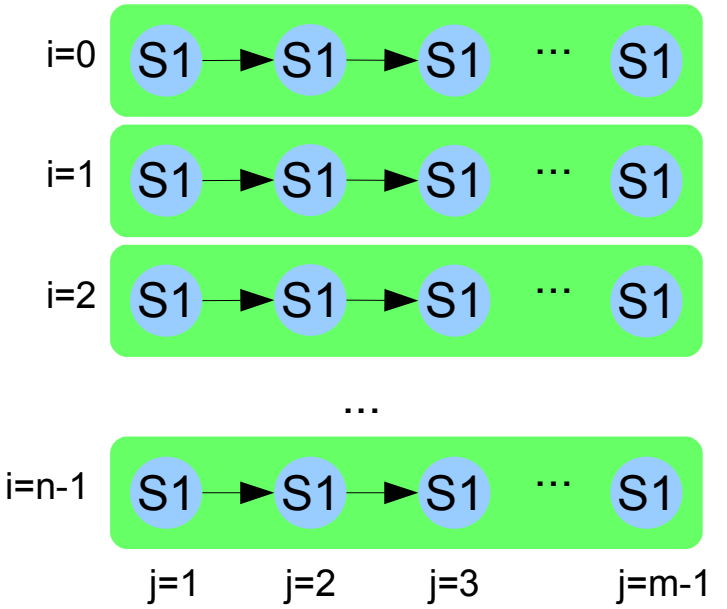
```
for (j=1; j<m; j++) {
  for (i=0; i<n; i++) {
S1  a[i][j] = a[i][j-1] + b[i];
  }
}
```

```
for (i=0; i<n; i++) {
  for (j=1; j<m; j++) {
S1  a[i][j] = a[i][j-1] + b[i];
  }
}
```

Parallelize the inner loop

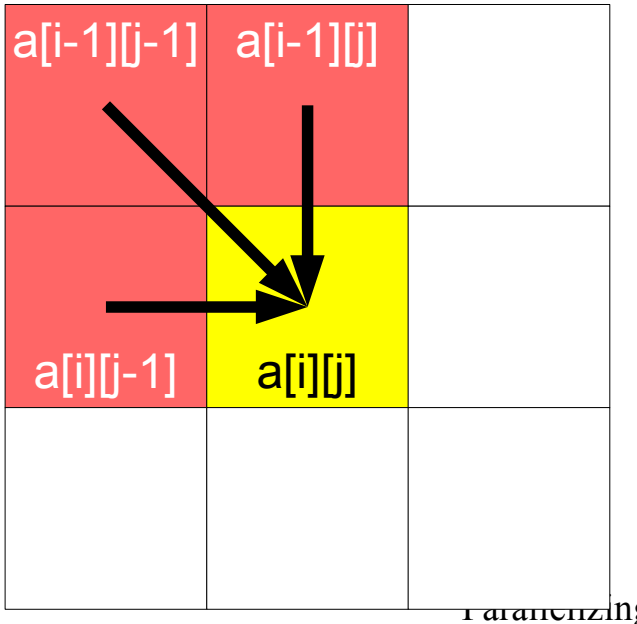


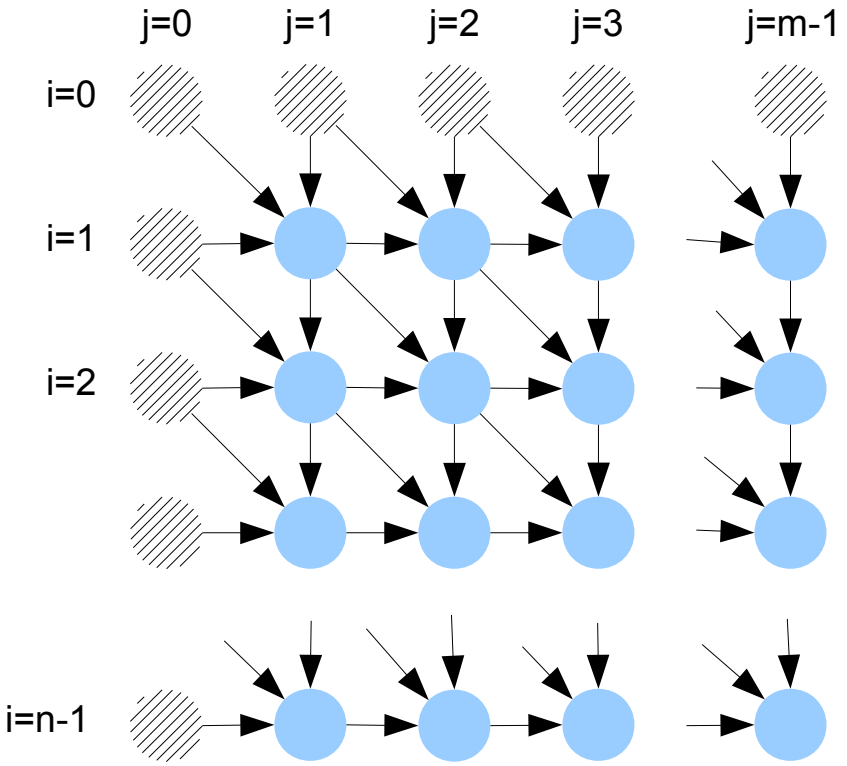

Parallelize the outer loop



# Difficult dependences

```
for (i=1; i<n; i++) {
  for (j=1; j<m; j++) {
    a[i][j] = a[i-1][j-1] +
              a[i-1][j  ] +
              a[i  ][j-1];
  }
}
```

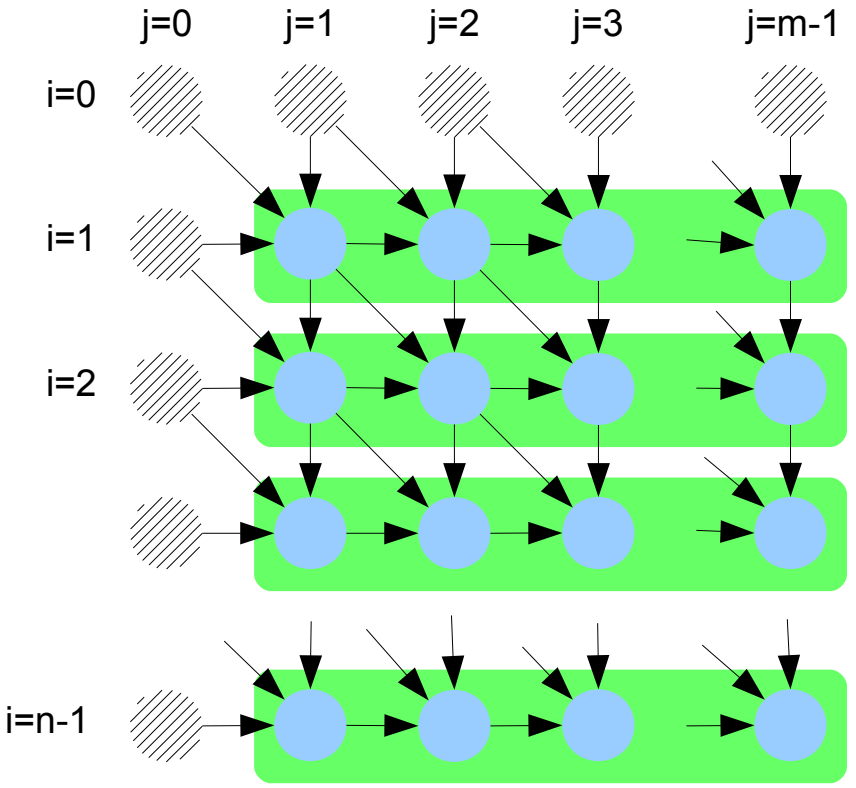


# Difficult dependences

- It is not possible to parallelize the loop iterations no matter what loop we consider

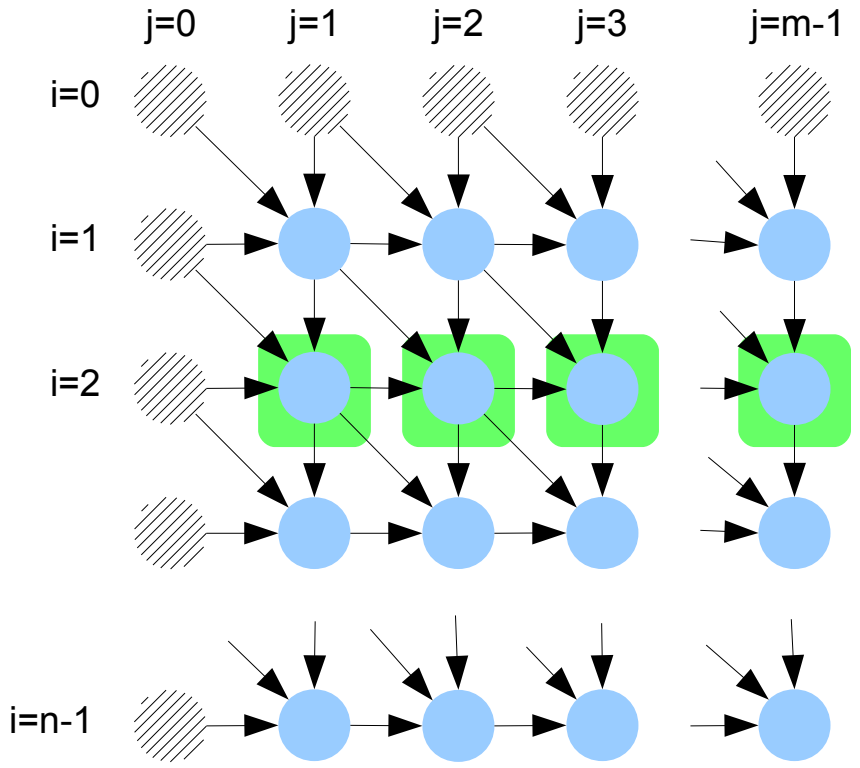
```
for (i=1; i<n; i++) {
  for (j=1; j<m; j++) {
    a[i][j] = a[i-1][j-1] +
              a[i-1][j  ] +
              a[i  ][j-1];
  }
}
```



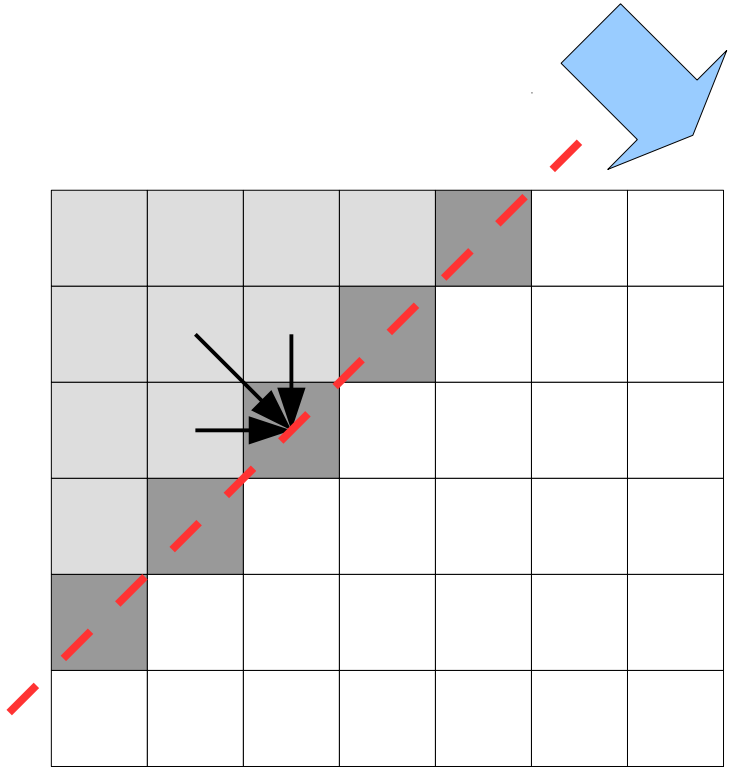
# Difficult dependences

- It is not possible to parallelize the loop iterations no matter what loop we consider

```
for (i=1; i<n; i++) {
  for (j=1; j<m; j++) {
    a[i][j] = a[i-1][j-1] +
              a[i-1][j  ] +
              a[i  ][j-1];
  }
}
```



# Solution



- It is possible to parallelize the inner loop by sweeping the matrix diagonally
  - *Wavefront sweep*

```
for (slice=0; slice < n + m - 1; slice++) {
    z1 = slice < m ? 0 : slice - m + 1;
    z2 = slice < n ? 0 : slice - n + 1;
    /* The following loop can be parallelized */
    for (i = slice - z2; i >= z1; i--) {
        j = slice - i;
        /* process a[i][j] … */
    }
}
```



https://stackoverflow.com/questions/2112832/traverse-rectangular-matrix-in-diagonal-strips

# Conclusions

- A loop can be parallelized if there are no dependencies crossing loop iterations
- Some kinds of dependencies can be eliminated by
  - Applying code transformations (reordering, aligning)
  - Using patterns (e.g., scans, reductions)
  - Sweeping across iterations "diagonally"
- Unfortunately, there are situations where dependencies can not be removed, no matter what